Cal Poly Department of Mathematics

Puzzle of the Week

Sep 24 - 30, 2010

Let m, b be positive real numbers. The line $y = mx + b$ intersects the parabola $y = x^2$ in two points, say A and B. Find the point P on the parabola, lying between A and B, such that the triangle ABP has maximal area.

Solutions should be submitted to Morgan Sherman:

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before next Friday. Those with correct and complete solutions will have their names listed in next week's email announcement. Anybody is welcome to make a submission.

Solution:

The point has coordinates $(\frac{m}{2}, \frac{m^2}{4})$. Here is a quick argument to find the solution. Let ℓ be the line $y = mx + b$. By taking the base of the triangle to be the line segment AB we see that to maximize the area we need to find the point P which is furthest from the line ℓ. Suppose $P = (x, x^2)$ is this point and consider the line ℓ' passing through P and parallel to ℓ. Then ℓ' must be tangent to the parabola, otherwise P would not be the furthest point from ℓ. Therefore $2x = m$ and we find the solution above.