I. Introductory Concepts

1.1 Why study measurement?

1.2 Introductory Examples

Example: Measuring your weight: “The measurement is not the thing.”

1.3 Some Important Definitions

We’ll start by defining some terms. Some of them will seem simple, but the following terms are so confused and misused by engineers (and non-engineers) that we need to spend some time describing them. From now on, we will use these terms carefully!

1.3.1 Accuracy

The most basic definition of accuracy is: how close a measurement is to the true value. An accurate instrument, therefore, represents the thing you are trying to measure closely.

1.3.2 Error

Error is the difference between the measured value and the true value. We can express error several ways:

**Absolute Error:** difference between measured value and true value of a variable, or

\[ \varepsilon = \text{measured value} - \text{true value} \]  \hspace{1cm} (1.1)

**Percent Error:**

\[ \%\text{error} = \left( \frac{\text{measured value} - \text{true value}}{\text{true value}} \right) \times 100\% \]  \hspace{1cm} (1.2)

Note that the sign is retained, which conveys information: a percent error of -2%, for example, means that the measurement is 2% below the true value.

**Percent Difference:**

\[ \% \text{ difference} = \left( \frac{\text{measurement 1} - \text{measurement 2}}{\text{measurement 2}} \right) \times 100\% \]  \hspace{1cm} (1.3)

• What’s the difference between percent difference and percent error?
• How do you decide which measurement is measurement 2?

1.3.3 Two Important Points Regarding Error

First, error does not mean mistake. You might be thinking of the term human error, but this is not an engineering measurements term. Second, and perhaps more importantly, error is usually not known. Why? Because the true value is not usually known. If the true value were known, why are we bothering...
1.3.4 Bias and Precision

Since we are talking about error, there are two general kinds of error. Some error is fixed, like if you weigh yourself on a scale that you know measures five pounds heavy. This fixed error is called bias. Technically, engineers refer to a measuring device as accurate if it has low bias error.

But there is second kind of error. For example, if you weigh yourself ten times in a row, and get a slightly different reading each time, the random variation in the measurement is referred to as precision. (Side note: precision, or random error, could be seen as you weigh yourself repeatedly with the same scale; this kind of precision is also referred to as repeatability. But if you weigh yourself with different scales, the degree to which these different scales agree is called reproducibility.) A graphical depiction of these terms is presented in Figure 1.1.

![Figure 1.1. Bias and precision as depicted in a plot.](image)

The key difference between bias and precision is that bias can be corrected; if you know the scale reads five pounds heavy, you can subtract that from your reading. Or, perhaps you can adjust the scale, or recalibrate it. Precision, on the other hand, is random; trying to characterize the random variation in measurements requires a course in Statistics. Fortunately, that is what this course is about!

1.3.5 Accuracy vs. Precision

People often misuse and confuse the terms accuracy and precision. First, remember that accuracy is related to the lack of bias (fixed) error in a measurement, while precision has to do with the randomness in repeated measurements. But we will go further, and invoke a classic analogy from nearly every textbook on measurement; we’ll call it “the target analogy.” Consider the targets shown in Figure 1.2. In this analogy, the bullseye represents the true value, and the holes represent our measurements. In the first target, the spread of the bullet holes (what shooters call the “pattern”) represents the precision of the measurement, and the spread is fairly large. In addition, the hole pattern isn’t centered over the bullseye; that distance is the bias, or lack of accuracy. So, the measurement is neither precise nor accurate.
In the second target, the precision has improved, but because of the bias that remains, the measurement is still not accurate. The third target is the opposite, where the measurement is accurate but not precise. Finally, the best case is one with high precision and low bias, and is this accurate and precise.

Figure 1.2. The target analogy for accuracy (lack of bias) and precision.

1.3.6 Resolution vs. Precision

These two terms get confused perhaps the most of all measurement terminology. For example, consider the digital temperature gauge depicted in Figure 1.3. By merely looking at the gauge, you’d be tempted to say that the gauge is *accurate* to one hundredth of a degree Fahrenheit (0.01°F), but that statement is false! The truth is, when simply looking at a gauge, you can only tell its resolution. You don’t know if it’s accurate, unless you have its calibration information (or you calibrated it yourself). By the way, gauges are frequently less accurate than their display’s resolution would imply! So, in short, you should never assume that a device is as accurate as its resolution implies.

So, we’ve decided that the statement that the gauge is “accurate to 0.01°F” is incorrect (or at least ignorant, or misleading). But can we say that the gauge is “precise to 0.01°F?” Again, the answer is no, this time because resolution and precision are not the same.

We already know that precision refers to the randomness in repeated measurements. Resolution refers to the fineness of a measurement, meaning the small division or change in a measurement can be detected. So, the resolution of the instrument is 0.01°F. The fact is, if we take repeated measurements, we might find that the temperature fluctuates by more than merely its resolution.

However, these two terms are linked: if the random variation in temperature were smaller than the resolution of the instrument (or stated differently, if the precision were higher), then the instrument’s reading would never change. That is, you’d not be able to detect precision better than the instrument’s resolution. So, the limit of an instrument’s precision is its resolution.
1.3.7 Uncertainty

As we said earlier, normally the error, defined by Eq. (1.1), is not known, and can only be estimated. We refer to this estimate of the error as the uncertainty in the measurement, which is usually presented as a range of values about the nominal value:

\[ x = (\text{nominal value}) \pm U_x, \quad (1.4) \]

where \( U_x \) is the uncertainty in \( x \). Now, uncertainty can take on several formats:

**Absolute Uncertainty:** The uncertainty of a measurement expressed in the units of the measurement. (Example: 120 lbm ± 3 lbm, or more simply, 120 ± 3 lbm)

**Percent Uncertainty:** The uncertainty expressed as a percentage of the nominal value. (Example: 120 lbm ± (3 lbm)/(120 lbm) \times 100\% \rightarrow 120 lbm ± 2.5%.

**Relative (or Fractional) Uncertainty:** The uncertainty expressed as a fraction of the nominal value. (Example: 120 lbm ± (3 lbm)/(120 lbm) \rightarrow 120 lbm ± 0.025 lbm/lbm. Note that the uncertainty is unitless, but the units (lbm/lbm) have been included anyway to make sure the audience knows what the value represents.)

**NOTE:** uncertainty and error are not the same thing! The terms are NOT interchangeable, although even engineers use error when they mean uncertainty. Never do this. (What IS the difference between error and uncertainty?)

1.3.8 Practical Sources of Measurement Uncertainty

Example: Measuring the temperature of this room.

**Reading Uncertainty** (sometimes called minimum or resolution uncertainty)

\[ u_{reading} = \pm \frac{1}{2} (\text{resolution}) \quad (1.5) \]

**Instrument** (or calibration) uncertainty
Statistical (or precision) uncertainty
Due to transient variations, spatial variations

Combining uncertainties: “root-sum-square” (summation in quadrature)

\[ u_{tot} = \pm (u_1^2 + u_2^2 + u_3^2 + \ldots)^{1/2} \]  

(1.6)

1.4 Other Definitions

1.4.1 Variable: Any quantity that can be measured (e.g., temperature, weight) or observed (roll of a die, number of students)

a. Dependent Variable: affected by changes of one or more variables (e.g., the temperature in a room is affected by the time of day; the temperature is therefore the dependent variable)

b. Independent Variable: can be varied independent of other variables (e.g., the time of day is independent of the temperature in a room)

c. Discrete Variable: possible values can be enumerated (e.g., roll of a die, number of students)

d. Continuous Variable: possible values are infinite (e.g., all physical properties, such as temperature, density, length)

e. Controlled Variable: held at a prescribed value during a measurement (e.g., temperatures were measured every 10 seconds: time is therefore a controlled variable)

f. Extraneous Variable: not controlled during an experiment

g. Random Variable: contains random scatter

1.4.2 Parameter: a functional relationship between variables (e.g., drag coefficient)

1.4.3 Noise: variation in a measured signal due to random fluctuations of extraneous variables

1.4.4 Interference: variation in a measured signal due to deterministic variation in extraneous variables (e.g., electrical interference)

1.4.5 Sequential Test: experiment where the controlled variable is varied in order

1.4.6 Random Test: experiment where the controlled variable is varied randomly

1.4.7 Repetition: repeated measurements made from the experiment, to examine the variability in observations for a single condition

1.4.8 Replication: duplication of the experiment under similar operating conditions (e.g. same test, different day)

1.4.9 Concomitant Methods: different methods for measuring the same variable; best if measurement technique is based on different physical properties

1.4.10 Static Measurement: one in which input variable is constant with time

1.4.11 Dynamic Measurement: one in which input variable changes with time
1.5 A Primer on Dimensions and Units

1.5.1 Dimensions versus Units

Nearly every engineering problem you will encounter will involve dimensions: the length of a beam, the mass of a concrete block, the time and velocity of an object’s fall, the force of the air resistance on an airplane, and so forth. We express these dimensions using specific units: for example, length can be expressed in feet, mass as kilograms, time as minutes, velocity as miles per hour, and force as newtons.\(^1\)

The goal of this section is to explain the use of dimensions and units in engineering calculations, and to introduce a few of the standard systems of units that are used.

1.5.2 How Dimensions Relate to Each Other

Dimensions (as well as units) act just like algebraic symbols in engineering calculations. For example, if an object travels 4 feet in 10 seconds, we can calculate its (average) velocity. First, algebraically:

\[ V_{\text{average}} = \frac{d}{t}, \]

where \( v \) is the symbol for velocity, \( d \) for distance, \( t \) for time. Plugging in the actual values (and units),

\[ V_{\text{average}} = \frac{(4 \text{ ft})}{(10 \text{ s})} = 0.4 \frac{\text{ft}}{\text{s}}. \]

Thus we can see that velocity in this case has the units feet per second (ft/s). We can convert feet to whatever we like: meters, miles, etc. We can also convert seconds to minutes, hours, days, etc. But the dimensions are always the same:

\[ \text{velocity} = \frac{[\text{length}]}{[\text{time}]} . \]

There are two kinds of dimensions: (1) primary dimensions, like length and time, and (2) secondary dimensions, like velocity, which are combinations of primary dimensions.

Because any given system of units we use has so many different measurements, standard units have been developed to make communication (and science and commerce) easier. We will explore three of these standard systems: the SI system, the British Gravitational system, and the English Engineering System. There are more!

1.5.3 The SI system

The SI (Système International d’Unités) system is the official name for the metric system. The system is described as an “MLtT” system, because its primary dimensions are mass (M), length (L), time (t), and temperature (T). The standard units are listed below.

<table>
<thead>
<tr>
<th>Primary Dimension</th>
<th>Standard Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass (M)</td>
<td>kilogram (kg)</td>
</tr>
<tr>
<td>length (L)</td>
<td>meter (m)</td>
</tr>
<tr>
<td>time (t)</td>
<td>second (s)</td>
</tr>
<tr>
<td>temperature (T)</td>
<td>Kelvin (K)</td>
</tr>
</tbody>
</table>

\(^1\) Notice that newtons is not capitalized. It is standard not to capitalize the name of the unit, even though the unit abbreviation is capitalized (i.e., N). It’s a confusing rule.
Secondary units are derived from these primary units. For example, velocity has units of m/s, acceleration is m/s², and force has units of…?

How do we relate force to the primary units? Isaac Newton discovered that the force on an object is proportional to its mass times its acceleration:

\[ F \propto ma \quad . \]

If we plug dimensions into the above relation, we see that

\[ \text{Force} \propto [M][L][t]^{-2} \quad . \]

Or, if we use primary SI units, we see that

\[ \text{Force} \propto \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \quad . \]

In honor of Newton, it was decided to give this particular set of terms the name newton (N). It is defined as

\[ 1 \text{N} \equiv 1 \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \quad . \quad (1.7) \]

So the unit of force in the SI system is the newton (N), defined as “the force required to accelerate a mass of 1 kg to an acceleration of 1 m/s².” Why not 2 kg? Or 10 m/s²? Actually, the number is arbitrary, but the number 1 is chosen for convenience.

**Example 1-1.** An object has a mass of 80 kg. If the acceleration of gravity is 9.81 m/s², what is its weight?

**Solution:** The weight of an object is the force of gravity on the object, which is given by

\[ W = mg \quad . \]

Plugging in values (and units) for m and g,

\[ W = (80 \text{ kg})(9.81 \text{ m/s}^2) \quad . \quad (a) \]

As you can see, the result of the above calculation does give us the correct dimensions and units for force. But for convenience, we know by definition that

\[ 1 \text{N} = 1 \text{kg} \cdot \text{m/s}^2 \quad . \]

Notice that we can manipulate the above equation slightly: If we divide both sides by 1 kg·m/s², we get

\[ \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} = 1 \quad . \]
Thus, if we multiply the right-hand-side of Equation (a) by the ratio above, we are merely multiplying by one – and a unitless value of one – which doesn’t change anything:

$$W = (80 \text{ kg})(9.81 \text{ m/s}^2) \left( \frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right).$$

Note that all the units cancel except for N, which yields

$$W = 784.8 \text{ N}.$$ 

Comments:

1. Note that we just used the definition of a newton as a kind of “conversion factor” to convert the answer above into a more convenient form. To be honest, it’s not necessary to use newtons, and in fact some engineers leave the units of force as kg·m/s² sometimes, because they know the units will cancel later. But just remember that you want to express your final answer in as relatable units as possible, for your audience’s understanding.

2. Recall that we determined the gravitational force by the equation

$$W = mg.$$ 

Why didn’t we use Newton’s second law, $F = ma$, where $a = g$? Isn’t that the same? Absolutely not! GRAVITY IS NOT ACCELERATION. IT IS A FORCE (PER UNIT MASS). It only looks like acceleration because it has units like that of acceleration (In fact, dimensionally, acceleration and force per unit mass are the same). Think about this. What is the force of gravity acting on your body right now? Are you in motion right now? If you are sitting still, you are not accelerating (relative to the ground). Then $a=0$! So is the force on your body zero? No!

Remember that in stating Newton’s second law, $F$ is the net force acting on the mass $m$. If the mass is stationary, the net force is zero. That is, the force of gravity on your body is exactly balanced by the force of the ground pushing up on you. You are in equilibrium, and therefore your acceleration is zero.

1.5.4 The British Gravitational System (“Slug” System)

The British Gravitational system of units is referred to as an “FLtT” system, because the primary dimensions are force (F), length (L), time (t), and temperature (T). The standard units are:

<table>
<thead>
<tr>
<th>Primary Dimension</th>
<th>Standard Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Force (F)</td>
<td>pound-force (lbf)</td>
</tr>
<tr>
<td>length (L)</td>
<td>foot (ft)</td>
</tr>
<tr>
<td>time (t)</td>
<td>second (s)</td>
</tr>
<tr>
<td>temperature (T)</td>
<td>Rankine (R)</td>
</tr>
</tbody>
</table>

If force is a primary dimension, how do we find the unit of mass? Mass is now a secondary dimension; we have to derive it. Newton’s second law always holds:

$$F \propto ma.$$ 

or, dimensionally,

$$[F] \propto [\text{mass}] \frac{[L]}{[t]^2}.$$ 

1-8
If we use primary units, we see that

$$\text{lb}_f \propto \frac{\text{mass} \cdot \text{ft}}{\text{s}^2},$$

Rearranging the above,

$$\text{mass} \propto \frac{\text{lb}_f \cdot \text{s}^2}{\text{ft}}.$$

We need a name for the unit of mass. Let’s call it a slug! Then we’ll define it by

$$1 \text{ lb}_f \equiv 1 \frac{\text{slug}}{\text{s}^2} \cdot \text{ft}. \quad (1.8)$$

We can interpret the above by saying, “one pound-force is the force required to accelerate 1 slug to an acceleration of 1 ft/s².” Again, we could have defined the slug as 10 lb·s²/ft, or 936.1 lb·s²/ft, but for the sake of simplicity, we choose 1 as the constant.

**Example 1-2.** An object has a mass of 5.59 slugs. What is its weight in Earth’s gravity?

**Solution:** As in Example 1, the weight of the object can be determined by

$$W = mg.$$

Substituting the mass and the value of standard Earth gravity, 32.174 ft/s², into the above,

$$W = (5.59 \text{ slug})(32.174 \text{ ft/s}^2)$$

The units above are not useful as units of force. But we know by definition that 1 slug = 1 lb·s²/ft, or

$$\left( \frac{1 \text{ lb}_f \cdot \text{s}^2/\text{ft}}{1 \text{ slug}} \right) = 1.$$

Multiplying the weight by the above gives

$$W = (5.59 \text{ slug})(32.174 \text{ ft/s}^2) \left( \frac{1 \text{ lb}_f \cdot \text{s}^2/\text{ft}}{1 \text{ slug}} \right)$$

$$= 179.85 \text{ lb}_f.$$

We see that the units in the above relation cancel, leaving the more convenient units of force.
1.5.5 The English Engineering System (“Pound-Mass System”)

In the English Engineering system of units, the primary dimensions are are force (F), mass (M), length (L), time (t), and temperature (T). Therefore this system is referred to as a “FMLtT” system. The standard units are shown below:

<table>
<thead>
<tr>
<th>Primary Dimension</th>
<th>Standard Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Force (F)</td>
<td>pound-force (lbf)</td>
</tr>
<tr>
<td>mass (M)</td>
<td>pound-mass (lbm)</td>
</tr>
<tr>
<td>length (L)</td>
<td>foot (ft)</td>
</tr>
<tr>
<td>time (t)</td>
<td>second (s)</td>
</tr>
<tr>
<td>temperature (T)</td>
<td>Rankine (R)</td>
</tr>
</tbody>
</table>

In this system, force and mass are primary dimensions. They must still be related by Newton’s second law:

\[ F \propto ma \]

or, dimensionally,

\[ [F] \propto [\text{mass}][L][t^2] \]

If we use the primary English units, we see that

\[ \text{lb}_f \propto \frac{\text{lb}_m \cdot \text{ft}}{s^2} \]

We don’t need to define a new unit, but we need to determine a constant in order to make the above relation exact. Let’s use 32.174! Then the relationship between pound-force and pound-mass is as follows:

\[ 1 \text{ lb}_f \equiv 32.174 \frac{\text{lb}_m \cdot \text{ft}}{s^2} . \] (1.9)

So in words, “one pound-force is the force required to accelerate one pound-mass to 32.174 ft/s^2.” Why 32.174? Because that just happens to be the value for the acceleration of gravity, \( g = 32.174 \text{ ft/s}^2 \). This value was chosen so that if an object has a mass of 10 lbm, its weight on the Earth will also be 10 lb. This “convenience” will become apparent later in one of the examples which follow. One final note: If we compare Equation (3) with Equation (2), we see that slugs and pounds-mass are related by

\[ 1 \text{ slug} = 32.174 \text{ lb}_m . \] (1.10)

**Example 1-3.** An object has a mass of 180 lbm. What is its weight in Earth’s gravity?

**Solution:** Again, the weight is given by

\[ W = mg , \]

which becomes

\[ W = (180 \text{ lb}_m)(32.174 \text{ ft/s}^2) . \]
To convert the units in the above equation into useful force units, we note that by definition, 1 lbf = 32.174 lbm-ft/s². Or,

\[
\left( \frac{1 \text{ lbf}}{32.174 \text{ lbm} \cdot \text{ft/s}^2} \right) = 1.
\]

Multiplying this constant with the weight gives

\[
W = (180 \text{ lbm})(32.174 \text{ ft/s}^2) \left( \frac{1 \text{ lbf}}{32.174 \text{ lbm} \cdot \text{ft/s}^2} \right) = 180 \text{ lbf}.
\]

Comment: Note that in Earth’s gravity, and the “pound-mass system,” the values of mass and weight are the same! In fact, that’s how the relationship between lb and lbm was defined. Remember, though, that the units represent different dimensions: lbf represents force, while lbm represents mass. So it is NEVER acceptable to write “1 lbf = 1 lbm.” This is not dimensionally correct; it is like saying that “1 apple = 1 orange.”

1.5.6 The Proportionality Constant gc

As a final note, if you haven’t yet heard of gc (“g sub c”) in your studies, you might. It’s sometimes referred to as the gravitational constant, and it is a less-common (some may say it’s obsolete, or old-fashioned) way to deal with the force-mass units relationship. DON’T USE THIS. But if you run across this term, how does it work?

Did you notice that, in every example above, we had to multiply the weight we calculated by a “conversion factor” to make the units come out right? Well, what some people do is just employ a factor, called gc, directly in the equation they are using. For example, Newton’s second law could be written as

\[
F = \frac{ma}{g_c}.
\]

Similarly, the gravitational force could be written as

\[
W = \frac{mg}{g_c}.
\]

Comparing gc in the equation above with the “conversion factors” we used in the examples, you can show that

\[
g_c = \frac{1 \text{ kg} \cdot \text{m/s}^2}{\text{N}} \quad \text{(SI system)},
\]

\[
g_c = \frac{1 \text{ slug} \cdot \text{ft/s}^2}{\text{lbf}} \quad \text{("slug" system)},
\]

and

\[
g_c = 32.174 \frac{\text{lbm} \cdot \text{ft/s}^2}{\text{lbf}} \quad \text{("pound-mass" system)}.
\]
Again, DON’T USE the \( g_c \) approach in your calculations. This technique can be confusing because you have to remember when you have to include \( g_c \) in your general equation. But, as you can see from the example problems, we ignored \( g_c \) entirely; as long as you ALWAYS keep track of ALL your units, you will know when you need to perform unit conversions in order to cancel certain units. Think of the definitions (1.7), (1.8), and (1.9) as a wild card that you can insert into a calculation when you need to simplify the units.

A summary of the basic unit systems is presented in Table 1.1. Memorize the force-mass relationships, and always use them explicitly in your calculations.

I can’t over-emphasize this point: NEVER DO UNIT CONVERSIONS IN YOUR HEAD. Always show them, no matter how trivial. Incorrect units are a leading cause of mistakes in calculations, sometimes leading to tragic results. Being explicit with your unit calculations will help you catch your own mistakes, and help your audience understand your calculations (and convince them that you know what you are doing).

<table>
<thead>
<tr>
<th>System</th>
<th>SI (“Metric” system)</th>
<th>British Gravitational (“slug” system)</th>
<th>English Engineering (“pound-mass” system)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary Dim’s</td>
<td>MLtT</td>
<td>FLtT</td>
<td>FMLtT</td>
</tr>
<tr>
<td>Mass</td>
<td>kg</td>
<td>slug</td>
<td>lbm</td>
</tr>
<tr>
<td>Length</td>
<td>m</td>
<td>ft</td>
<td>ft</td>
</tr>
<tr>
<td>Force</td>
<td>N</td>
<td>lb(_f)</td>
<td>lb(_f)</td>
</tr>
<tr>
<td>Time</td>
<td>s</td>
<td>s</td>
<td>s</td>
</tr>
<tr>
<td>Temperature</td>
<td>K</td>
<td>R</td>
<td>R</td>
</tr>
<tr>
<td>Force-Mass Relationship</td>
<td>( 1 \text{N} \equiv \frac{1 \text{kg} \cdot \text{m}}{\text{s}^2} )</td>
<td>( 1 \text{lb}_{f} \equiv \frac{1 \text{slug} \cdot \text{ft}}{\text{s}^2} )</td>
<td>( 1 \text{lb}<em>{f} \equiv 32.174 \frac{\text{lb}</em>{m} \cdot \text{ft}}{\text{s}^2} )</td>
</tr>
<tr>
<td></td>
<td>( g_c = \frac{1 \text{kg} \cdot \text{m/s}^2}{\text{N}} )</td>
<td>( g_c = \frac{1 \text{slug} \cdot \text{ft/s}^2}{\text{lb}_{f}} )</td>
<td>( g_c = 32.174 \frac{\text{lb}<em>{m} \cdot \text{ft/s}^2}{\text{lb}</em>{f}} )</td>
</tr>
</tbody>
</table>

Table 1.1. Summary of Unit Systems
Unit examples (solutions on the next page):
Example 1-4. The pressure acting on a 1.25 in² test specimen equals 15 MPa. What is the force (in N) acting on the specimen?

Example 1-5. The weight of a large steel cylinder is to be computed from measurements of its diameter and length. Let its length L be equal to 3.32 m and its diameter d equal to 0.3605 m. Suppose that the density of the steel equals 7835 kg/m³. Calculate the weight of the cylinder (in N) and report your result in a clear and unambiguous form.
Solutions

Example 1-4. The pressure acting on a 1.25 in² test specimen equals 15 MPa. What is the force (in N) acting on the specimen?

\[ P = \frac{F}{A} \]

\[ F = PA \]

\[ = (15 \text{ MPa}) (1.25 \text{ in}^2) \left( \frac{2.54 \text{ cm}^2}{\text{in}^2} \right) \left( \frac{1 \times 10^6 \text{ N/m}^2}{1 \text{ MPa}} \right) \left( \frac{1 \text{ m}}{100 \text{ cm}} \right)^2 \]

= 12096.75 N

= 12000 N (to two sig. figs.) Answer

Example 1-5. The weight of a large steel cylinder is to be computed from measurements of its diameter and length. Let its length \( L \) be equal to 3.32 m and its diameter \( d \) equal to 0.3605 m. Suppose that the density of the steel equals 7835 kg/m³. Calculate the weight of the cylinder (in N) and report your result in a clear and unambiguous form.

\[ W = Mg \]

\[ W = \rho V g \]

\[ V = \frac{\pi d^2 L}{4} \]

\[ W = \left( 7835 \frac{\text{kg}}{\text{m}^3} \right) \left( 0.3605 \text{ m} \right)^2 \left( \frac{3.32 \text{ m}}{4} \right) \left( 9.81 \frac{\text{m}}{\text{s}^2} \right) \left( \frac{1 \text{ N}}{1 \text{ kg m/s}^2} \right) \]

= 26019.7...

= 26000 N (three sig. figs.) Answer

Because of...
1.6 Significant Figures

1.6.1 What Are Significant Figures, and Why Do We Care About Them?

Significant figures is a way to express real measurements or calculated values as precisely as the values deserve. For example, if someone tells you that the mass of an object is 32.4 kg, they presumably decided that reporting the value to any more decimals, as in 32.43756 kg, overstates or misrepresents the precision of the actual measurement. Perhaps they know that the mass scale, no matter many decimals it displays, is only accurate to a tenth of a kilogram.

Now, you may not have seen the topic of significant figures in your coursework. In fact, most of your science and engineering courses practically ignore the topic. Why? Because those courses are focused more on developing theory, which is hard enough; the last thing they want to do is add to it the complexity of dealing with real numbers with measurement uncertainties. And if you have seen significant figures before, it was likely introduced in your Physics or Chemistry laboratories, because these laboratories work with real data.

We can’t overstate the importance of being honest and accurate when reporting experimental data. In fact, one of the first questions you ask every time you report data or calculations is, “Are the significant figures correct? Or am I reporting too many digits, and therefore lying about the precision of my results?”

So in summary, there are two reasons why we care about significant figures:

1. Significant figures are used because we don’t want to overstate or misrepresent the precision of the measurements or calculations that we report to an audience.
2. The number of significant figures implies (at least roughly) the uncertainty of a measurement when the uncertainty isn’t explicitly stated.

1.6.2 What figures are significant?

Identifying the number of significant figures is not quite as simple as merely counting the digits, because not all digits have something to do with the precision or the uncertainty of a measurement. So, what follows are the rules for identifying which figures are significant in reported data:

Rules for Identifying Significant Figures in Reported Data

1. All non-zero numbers are significant.

   Examples:  
   a. The value 237 has three significant figures.
   b. The value 123.45 has five significant figures.

2. Zeros between non-zero numbers are significant.

   Examples:  
   a. The value 100.1 has four significant figures.
   b. The value 1027.0034 has eight significant figures.

3. Zeros used to locate the decimal point are NOT significant, because they define magnitude, not precision.
Examples: a. The value 0.0000543 has three significant figures. (The zeros are merely placeholders that define the order of magnitude.)  
b. The value $5.43 \times 10^{-5}$ has three significant figures. (And it’s the same value as in the previous example, just in scientific notation. Note that the exponent -5 only defines order of magnitude.)

4. **Zeros to the right of non-zero numbers are usually significant.**

Examples: a. The value 0.0230 has three significant figures.  
b. The value $1.200 \times 10^3$ has four significant figures.  
c. The value 100 has either one, two, or three significant figures.

Comment: The value 100 is ambiguous: we don’t know whether the zeroes are significant or merely placeholders. This problem can be avoided by reporting the value several ways, from worst to best:

- Some texts suggest underlining the least significant digit, as in 10̅0, which implies two significant figures. Since your audience probably doesn’t know that rule, avoid using it.
- Some texts suggest that including the decimal, i.e. “100.” implies that the all zeroes are significant. But this only works if all zeroes are indeed significant. Also, it relies on your audience to know that rule. So avoid it.
- Using scientific notation: $1.00 \times 10^2$ (it works every time, although scientific notation can be awkward to read and should be used only for very small or very large numbers).
- Using words: 100 to two significant figures. Clear and direct.
- Finally, if you do know the uncertainty, report it explicitly and remove all doubt as to how precise the measurement is. Example: 100±25.

5. **Some values are exact, and therefore not analyzed for significant figures.**

Examples: a. Some conversion factors are exact. The conversion 1 in = 2.54 cm is exact.  
b. There are 12 eggs in a dozen. This value is exact.

**Example 1-6.** Identify the number of significant figures in the following values: (a) 123.1, (b) 5.00, (c) $1.500 \times 10^3$ (d) 1900 (e) 0.002003

Solution: a. The value 123.1 has four significant figures, by Rule 1.  
b. The value 5.00 has three significant figures, by Rule 4.  
c. The value $1.500 \times 10^3$ has four significant figures, by Rule 4.  
d. The value 1900 is ambiguous; it has either two, three, or four significant figures.

To eliminate this ambiguity and imply three significant figures, this value could be written one of two ways:

- $1.90 \times 10^3$
- 1900 to three significant figures

e. The value 0.002003 has four significant figures, by Rules 2 and 3.
1.6.3 How Do Significant Figures Relate to Uncertainty?

We said that significant figures imply the uncertainty in a measurement or calculated value. Now we will attempt to assign a value to that uncertainty.

Let’s consider the example we began with: a reported mass of 32.4 kg. If we assume that the raw value contained more digits, it seems reasonable to assume that they rounded the raw value to obtain the reported value. That is, the true value is likely between $32.350 \pm 32.450$, because any number between these limits would get rounded to 32.4 kg. In other words, the true value could be anywhere $±0.05$ kg of the reported value of 32.4 kg. In summary, we obtained the following estimate for the uncertainty to be:

$$32.4 \text{ kg implies } 32.4 \pm 0.05 \text{ kg}.$$  

By the way, this approximation is the same as when we estimate the uncertainty in a digital measurement.

Now, remember that we don’t know the actual uncertainty in the measurement. This technique merely estimates the uncertainty. Admittedly, if the engineer had reported the actual uncertainty along with the measurement, e.g., 32.4 ± 0.03 kg, we wouldn’t need to estimate it! That said, this technique seems to be a reasonable approximation when uncertainties are not explicitly given. To summarize this rule:

**Rule for estimating the uncertainty in a value when none is given:**

Treat the number as if it had been rounded to the least significant digit, so the uncertainty is one-half the resolution. This is the same rule we follow for estimating the uncertainty in a digital readout.

1.6.4 How Do Significant Figures Work in Calculations?

We now know how to identify the significant figures in a measurement, and roughly estimate the uncertainty they imply. But what happens when we perform calculations with real measurements? For example, take the calculation $10.0/3.0 = 3.33333333333$. Given the precision of the values 10.0 and 3.0, it makes no sense to report all the digits that our calculator supplies.

It turns out that analyzing how measurement uncertainties propagate through a formula or series of calculations is not easy, and requires a calculus-based approach that we will learn later in the course. However, for very simple mathematic expressions, there are some rules that you would have seen in your Physics or Chemistry Labs. It’s worth reviewing these rules, since they give us some mathematical insight. But in Chapter 6 we will develop a more rigorous technique that applies to any mathematical calculation and will render the following rules largely obsolete.

**Rules for Significant Figures in Calculations**

1. **Addition or subtraction:** The resulting value should be reported to the same number of *decimal places* as that of the term with the least number of *decimal places*.

   Examples:  
   a. $5.6 + 161.032 = 166.6$ (166.632 rounded to one *decimal place*)  
   b. $152.11 \text{ – } 33.0343 = 119.08$ (119.0757 rounded to two *decimal places*)

   Why? When numbers are added or subtracted, it seems reasonable that their uncertainties would approximately add (even if the nominal values are subtracted, the
uncertainties would add... why?). Considering the first example, if the first number, 5.6, is only known to the tenths place, does it matter that the other value is known to the thousandths place? The uncertainty in the value 5.6 would overshadow the precision of the other number. In effect, the least accurate value wins.

2. **Multiplication or division:** The resulting value should be reported to the same number of [significant figures](#) as that of the term with the least number of [significant figures](#).

   Examples:  
   a. \(152.06 \times 0.24 = 36\) (36.4944 rounded to two [significant figures](#))  
   b. \(1.057 \times 10^3 / 19.5 = 54.2\) (54.205128... rounded to three [significant figures](#))

   Why? Consider the multiplication example. If two values are multiplied, it stands to reason that in some way their uncertainties also multiply. For example, if two numbers \(a\) and \(b\) are multiplied, but \(a\) is then increased by 10 percent, the result would be

   \[
   F = (0.1a)b = 0.1(ab).
   \]

   In other words, a percentage change in one of the values gives the same percentage change in the calculated result. How does that affect how report significant figures? You can probably imagine that the value with the smaller number of significant figures has the largest percentage uncertainty, and that therefore its uncertainty would dominate in the calculated result. If that explanation isn’t perfectly clear, don’t worry; focus on remembering the rule for now, and we’ll explore how it works when we get to Chapter 6.

3. **Averages and standard deviations:** It is customary to report them to one more decimal place than the original data.

   Example: The average of 34.3, 20.0, 77.8, and 16.6 is reported as 37.18.

   Why? It seems reasonable that if you averaged the values 1 and 2, that reporting the average as 1.5 instead of 2 would be more informative!

4. **Keep an extra digit or two during intermediate calculations.**

   Why? To avoid accumulating round-off error. For example, consider the expression:

   \[
   \frac{61.452 - 52.1}{13.5 - 12}.
   \]

   If we follow the rules of significant figures at each step, we would get:

   \[
   \frac{9.352}{1.5} \rightarrow \frac{9.4}{2} \rightarrow 4.7 \rightarrow 5.
   \]

   Now, had we not rounded at all, the calculator would report

   \[
   \frac{9.352}{1.5} \rightarrow 6.234666,
   \]

   which, if we then reported to the same significant figures, would yield a value of 6, not 5. Why didn’t the answers match? Remember that significant figures is an approximate way to estimate the uncertainty in a measurement. So, when we round
a value, we introduce even more error, since we know the values of the decimals we’re removing. The lesson here is that, if we strictly follow significant figures rules at every step of a calculation, we introduce error that could have been avoided. The solution is to carry (and write down!) an extra digit or two at every step, then reduce the final result (the “reported” answer) to the correct number of significant figures that you would have chosen had you followed the rules at each step.

1.6.5 How to Round Numbers

It’s hard to believe that a college-level textbook would contain a section on how to round numbers. But as it turns out with so many things we learned earlier in life, the real world introduces complexities that we as engineers have to deal with.

Rounding is a perfect example. The rule we learned was easy: if you want to round a value to a particular decimal place, you simply look at the very next decimal. If that digit is 0, 1, 2, 3, or 4, you round UP. If the digit is 5 or greater (including if the value is 5 but followed by any non-zero digits), you round DOWN (truncate, in other words).

But there’s a problem with that rule: a special case. If that next digit is exactly 5 (meaning no other digits follow other than zero), the unrounded number is exactly halfway between the rounded and truncated value. Take, for example, the number 123.45. If we wanted to round to the tenths place, the answer could be either 123.4 or 123.5, because again, the original value is exactly halfway between them.

So what should we do? We could simply follow the rule we stated above, and simply round the value UP, to 123.5. But you shouldn’t always round up, because that introduces a bias. Over many, many calculations (especially in computer code involving millions of computations), this bias can build up. In fact, there are examples where the accumulation of this rounding error has caused catastrophic failure.

So again, what should we do? Ideally, you should, on average, round up half the time, and round down half the time. And the simplest rule to apply is to look at the value of the decimal place you plan to round to: if that digit is even, leave it unchanged, and if the digit is odd, round it up. This technique eliminates the bias over many calculations. Finally, we can summarize the rounding rules as follows.
Rounding Rules

1. If the number to the right of the least significant digit is LESS THAN 5, round the least significant digit DOWN.

   Examples: 123455 rounded to three significant figures is 123000.
   \[1.98385 \times 10^5\] rounded to three significant figures is \[1.98 \times 10^5\].
   0.996265 rounded to three significant figures is 0.996.

2. If the number to the right of the least significant digit is GREATER THAN 5 (including 5, with any non-zero digits to the right of it), round the least significant digit UP.

   Examples: 123455 rounded to four significant figures is 123500 (because the 5th digit isn’t exactly 5, it’s greater than 5 because of the non-zero digit to the right)
   \[1.98385 \times 10^5\] rounded to four significant figures is \[1.984 \times 10^5\].
   0.996265 rounded to four significant figures is 0.9963.

3. SPECIAL CASE: If the number to the right of the least significant digit is EXACTLY 5 (no nonzero digits afterward), round the least significant digit UP if it is an ODD number; if it is EVEN, leave it unchanged.

   Examples: 123455 rounded to five significant figures is 123460.
   \[1.98385 \times 10^5\] rounded to five significant figures is \[1.9838 \times 10^5\].
   0.996265 rounded to five significant figures is 0.99626.

Example 1-7. Round the value 5195.48535 to each successive decimal place, one at a time.

Solution: Beginning with the largest decimal place, the rounded values are as follows:

\[5195.48535 \rightarrow 5195.4854\] (The value 3 was followed by exactly 5; since 3 is odd, it gets rounded up)

\[5195.48535 \rightarrow 5195.485\]
\[5195.48535 \rightarrow 5195.49\] (The value 8 gets rounded up, by Rule 1. Rule 3 does not apply because the first 5 is followed by nonzero digits)

\[5195.48535 \rightarrow 5195.5\]
\[5195.48535 \rightarrow 5195\]
\[5195.48535 \rightarrow 5200\]
\[5195.48535 \rightarrow 5200\]
\[5195.48535 \rightarrow 5000\]
Extra:
How Precise Can We Report Earth’s gravity?
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We use gravity \( g = 9.81 \text{ m/s}^2 \) or \( 32.2 \text{ ft/s}^2 \) often in engineering calculations, but since this course deals with accuracy, have you ever stopped to consider how accurate the value is? You might be surprised to read this, but the value you choose for gravity may not always be “correct,” depending on the accuracy you need for your application!

First of all, gravity is not a constant value. It varies along the surface of the earth, with elevation above the earth, and even with time. For the sake of simplicity, we usually use standard gravity\(^2\), defined as \(9.80665 \text{ m/s}^2\), or \(32.1740 \text{ ft/s}^2\). This value is not your local value – instead, it represents the mean gravity at 45º latitude, at mean sea level. This is the value we typically use in engineering calculations, although we usually round it down to one or two decimal places.

The gravity on the surface of the Earth varies due to a number of effects. The major effects are:

1. The rotation of the Earth. The rotation of the Earth reduces the force you feel at your feet (you would therefore feel lighter at the equator than at the poles of the Earth). Also, the rotation causes the Earth to be shaped like an oblate spheroid (flattened sphere – flatter at the poles), which means that the radial distance you are from the center of the Earth (and hence gravity) varies depending on where you are on the surface. Both of these effects vary with latitude. This variation is predictable, and is given by an equation known as the International Gravity Formula:\(^3\)

\[
g_0 = 9.7803269 \pm 0.0001986 \lambda^2 0.0061912 \lambda + 0.0000026 \lambda^3 + 0.00000001 \lambda^4,\]

where \( \lambda \) is the geographic latitude and \( g_0 \) is called normal gravity. The variation due to Earth’s rotation is on the order of \( \pm 0.03 \text{ m/s}^2 \).

2. Elevation above sea level. At higher elevations, you are further from the center of the Earth, so the Earth’s pull is less. How big is this effect? At an elevation of about 1000 m, gravity reduces by about \(0.0001 \text{ m/s}^2\).

3. Variation in mass. Gravity is a function of mass, and since the mass of the earth is not uniform, the gravity varies as well. The most accurate measurement of local gravity variation was performed by GRACE (Gravity Recovery and Climate Experiment), a satellite-based measurement. These measurements show that the local variation of gravity (relative to normal gravity) is on the order of about \( \pm 0.0006 \text{ m/s}^2 \) (See Figure 1).

\(^2\) Established by the 1901 International Conference on Weights and Measures (in French, the acronym is GCPM).
\(^3\) Established by the World Geodetic System 1984. There are earlier versions of this formula.
4. Tides. Tidal variation (due to the gravitational pull of the sun and moon) contributes to a variation of about ± 0.000003 m/s².

These effects are summarized below, illustrating the relative effect of these factors:

\[ g = 9.\text{XXXXXX} \text{ m/s}^2 \]

- Tides \( \approx \pm 0.000003 \text{ m/s}^2 \)
- Elevation (below 1000 m the correction is less than approximately -0.0001 m/s²)
- Local gravity variation \( \approx \pm 0.0006 \text{ m/s}^2 \)
- Latitude \( \approx \pm 0.03 \text{ m/s}^2 \)

**Bottom line:** If you do not account for latitude, a value for \( g \) of 9.8 m/s² or 9.81 m/s² is as accurate as you can claim. Correcting for latitude allows you to claim perhaps three decimal place accuracy. Beyond this, you would have to get better information on your local gravity variation.
References


Homework

Use engineering paper, and show all work. Work all problems in the unit system given (i.e., do not simply convert to SI, solve the problem in SI, and then convert back to the original units). You may need to review some topics in your Physics textbook to solve some of these problems.

1-1 You measure the mass of 10 M&M candies, and find the average mass to be 0.896 g with a statistical uncertainty of 0.025 g. The resolution of the scale is 0.001 g, and variation of temperature in the room adds an error of ± 0.005 g to the scale.

a. Find the total error (i.e., uncertainty) of the measurement of average mass.
b. Express the complete measurement (nominal value and uncertainty). Do this three ways: with absolute uncertainty, percent uncertainty, and relative uncertainty.

1-2 To check the accuracy of a mass scale, you place calibration mass of 10.000 g on the scale (for the purposes of this problem, the calibration mass is exact). Your scale reads 9.987 g. Determine the (absolute) error, percent error, and percent accuracy of the device at this reading.

1-3 Answer the following questions. Assume standard gravity in each case.

a. How much does 3.0 kg weigh on the Earth, in N?
b. How much does a person with a mass of 183 lbm weigh on the moon (lbf)? (The moon’s gravity is exactly one-sixth the Earth’s)
c. A gallon of water weighs about 8 lbf on Earth. What is its mass in slugs?

1-4 Convert the following, without using any special functions in your calculator.

a. –1.00 °C to degrees Fahrenheit
b. 103 °F to degrees Rankine
c. 237 K to degrees Celsius
d. A temperature difference of 10 degrees Celsius to degrees Kelvin.
e. A temperature difference of 10 degrees Rankine to degrees Fahrenheit.
f. A temperature difference of 10 °C to °F.

1-5 What is the pressure of water on the bottom of a 6.0 ft deep pool, (a) relative to the pressure at the surface (lbf/in²), and (b) absolute, if the pressure at the surface is atmospheric? Use the principles of fluid statics that you learned in Physics. Assume the density of water to be \( \rho_w = 62.4 \text{ lbf/ft}^3 \).

1-6 A barrel of water is weighed on a scale to be 150 lbf.

a. Neglecting the weight of the barrel, and assuming the water to have a density of \( \rho_w = 62.4 \text{ lbf/ft}^3 \), estimate the volume of the water.
b. To what extent does the buoyancy of the surrounding air affect the weight measurement? (Hint: estimate it using Archimedes Principle. Assume the air density to be $\rho_{\text{air}} = 0.0768 \text{ lbm/ft}^3$).

1-7 You may recall from Physics that the heat capacity, $C$, of a substance is the energy gained for a given temperature rise (units of Btu/°F, kJ/K, etc.). Specific heat, $c$, is the heat capacity per unit mass (Btu/lbm·°F, kJ/kg·K, etc.).

The following experiment has been designed to measure the specific heat, $c$, of water: Water with a mass of 1.00 lbm is heated by an electric heater that delivers heat at a rate of 300 Btu/hr (to 2 significant figures). Over a period of 15 minutes, the temperature of the water rises 75 °F. What is the specific heat of the water?

1-8 For the sake of conservation, you decide to measure how fast the water evaporates from your swimming pool. You do this by recording the level of the pool every day for a month, and calculating from it and the surface area the water evaporated. Answer the following questions:

   a. Name as many variables that may influence this measurement.
   b. Identify any dependent and independent variables
   c. Identify any discrete and continuous variables
   d. Identify any controlled and extraneous variables

1-9 Explain three concomitant methods by which you could determine the diameter of a roughly 2-inch diameter steel sphere. In terms of accuracy, what are the advantages and disadvantages of each method?