1. Arfken 7.2.5

2. In class we showed an infinite sum could be mapped into the complex plane and evaluated using the residue theorem. When analyzing a relativistic boson gas using high temperature field theory one encounters the infinite sum,

\[ \sum_{n=-\infty}^{\infty} \frac{1}{(2\pi n T)^2 + p^2}. \]

Evaluate this sum. (Hint: Consider the function cot(z/2T). Your answer should be \( \frac{1}{2T} \coth \left( \frac{p}{2T} \right) \). Be sure to justify your logic.)

3. Show \( \int_{0}^{\pi} (\sin \theta)^{3} d\theta \approx \sqrt{\frac{2\pi}{x}} \) for large x. (Hint?: \( 2 = e^{\ln 2} \))

4. Show that the leading asymptotic behavior of the Bessel function \( J_0(x) \) for large x is

\[ J_0(x) = \frac{1}{\pi} \int_{0}^{\pi} \cos(x \sin \theta) d\theta \approx \sqrt{\frac{2}{\pi x}} \cos \left( x - \frac{\pi}{4} \right). \]