ONE-DIMENSIONAL SYSTEMS: RC Circuit

This experiment is a continuation of our study of one-dimensional systems. All of these systems are governed by an equation of the form $\dot{x} = f(x)$, where $x$ is the dynamical variable. For this experiment $x$ is the charge on a capacitor.

Objectives:
1. Acquaintance with Data Logger software
2. Graphical Analysis:
   - description of system behavior from graph
   - identification of fixed points from plot of time dependence
   - identification of fixed points from phase diagrams
3. Explore the fixed points:
   - dependence on system parameters
   - independence of initial conditions
   - stability of fixed points

This experiment explores the discharging of a capacitor. The dynamical variable is the charge on the capacitor. The circuit comprises two components: a resistor and a capacitor. The battery is for charging. With the switch in position "a" the capacitor will become charged very quickly. With the switch in position "b" the capacitor is now connected to a resistor, the charge rushes through the circuit neutralizing the plates.

![Circuit Diagram]

The equation of motion for this system is

$$\dot{q} = -\frac{1}{RC} q$$

(Eqn. 1)

where $q$ is the charge, $R$ is the resistance and $C$ is the capacitance.

Actions:

The circuit is set up with two probes (voltage and current) connected to the computer. We examine the discharge by measuring the voltage across the capacitor which is proportional to $q$ and the current through the resistor which is by definition $I = \frac{dq}{dt} = \dot{q}$.

1- Choose Two Graphs from the Display item on the menu. You should see two graphs on the screen, choose port one and port 2 for the vertical axis label on one graph. (Voltage and current vs. time.) Then choose port 2 for the vertical axis label on the other and port 1 for the horizontal. (Phase plot.) Set the data rate to 10 pts/sec for each probe (choose Data Rate under the Collect item on the menu). Set the time axis from 0 to 30 seconds.
2-Put the leads on either side of the "5 Ω" mark for the resistor. Move the switch to position "a" to charge capacitor. Now move switch to "b" and immediately click "Start."

3-Once you have data choose reasonable values for the axes ranges. Then save this data by clicking on "Data A->Data B".

4-Change the resistance from "5 Ω" to "10 Ω" and take data again.

5- Print out the graphs of voltage and current vs. time and the phase plot for at least two different values of the resistor.

Questions:
1. Describe the pattern you see for \( q \) vs. \( t \) and the phase plot and explain why it changes as the resistance changes.
2. What is the fixed point for this system? Is it stable? Explain how you know and/or what experiment you would do to find out
3. What is the initial condition for this experiment? Does the location of the fixed point depend on its value? Explain how you know or how you would find out?
4. Describe a simple variation of this experiment that would change the location of the fixed point.
5. Is this a linear system? How does it compare to the cooling lab?