Homework #4

Reading Assignment: Chapter 5.1-5.6

Problems: 5.1 (Hint: use continuity equation to check for compressible flow behavior), 5.15, 5.32, 5.46 (Hint: substitute in $\theta = \pi$ and $r = R$ into velocity function before solving for accelerations; use the plots to verify locations of maximum and minimums), 5.47 (Plot accelerations at 0, 10, 20 and 30 seconds), 5.65

Answers: 5.1 (a) compressible, (b) compressible, (c) incompressible behavior
5.15 (a), (b), and (c) incompressible behavior
5.32 (-2.86 x 10^-2 $i$ - 2.86 x 10^-4 $j$) m/s^2, 0.0025
5.46 $a_r(r, \theta = \pi) = \frac{2 U^2 R^2}{r^3} \left[ 1 - \left( \frac{R}{r} \right)^2 \right]$, $a_\theta(r, \theta = \pi) = 0$, $a_r(r = R, \theta) = -\frac{4 U^2}{R} \sin^2 \theta$,
\[
a_\theta(r = R, \theta) = \frac{4 U^2}{R} \sin \theta \cos \theta
\]
5.47 $a_x = \frac{U_0}{2(1-bx)} \left\{-\omega \sin (\omega t) + \frac{b U_0 [1 + \cos (\omega t)]^2}{2(1-bx)^2} \right\}$
5.65 $0$, $-\frac{2 u_{max}^2 \gamma}{b^2}$, $\zeta = \frac{2 u_{max} \gamma}{b^2}$

Objectives:
1. Be able to derive the basic laws for a control volume in differential form and understand the physical significance of each term.
2. Be able to use vector notation and apply the operators in Cartesian coordinates and cylindrical coordinates (see handout on vector notation).
3. Be able to derive the following special cases of the basic laws for a control volume: one-dimensional flow, two-dimensional flow, incompressible, and steady state.
4. Be able to apply the basic laws for a control volume in differential form to solve for the velocity profile for Couette and Poiseuille flow.
5. Be able to calculate the motion of a fluid particle (kinematics) which includes translation (particle acceleration), rotation (vorticity), angular deformation (proportional to shear stress), and linear deformation (volume dilation rate).