Tacit collusion in the presence of cyclical demand and endogenous capacity levels

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A R T I C L E   I N   F O

Article history:
Received 26 May 2007
Received in revised form 9 February 2009
Accepted 18 July 2009
Available online xxxx

JEL classification:
L2
L4
D2
D4

Keywords:
Tacit collusion
Capacity constraints
Booms
Busts

A B S T R A C T

We analyze tacit collusion in an industry characterized by cyclical demand and long-run scale decisions; firms face determinist demand cycles and choose capacity levels prior to competing in prices. Our focus is on the nature of prices. We find that two types of price wars may exist. In one, collusion can involve periods of mixed-strategy price wars. In the other, consistent with the Rotemberg and Saloner (1986) definition of price wars, we show that collusive prices can also become counter-cyclical. We also establish pricing patterns with respect to the relative prices in booms and recessions. If the marginal cost of capacity is high enough, holding current demand constant, prices in the boom are generally lower than the prices in the recession; this reverses the results of Haltiwanger and Harrington (1991). In contrast, if the marginal cost of capacity is low enough, then prices in the boom are generally higher than the prices in the recession. For costs in an intermediate range, numerical examples are calculated to show specific pricing patterns.

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1. Introduction

Inferring collusion is difficult because there rarely is a “smoking gun.” Instead, empirical research has focused on dynamic pricing patterns predicted by theories of collusion that are inconsistent with static models of competition. One such strand of literature tests whether observed equilibrium prices are consistent with the predicted collusive prices when firms face cyclical demand. This literature leverages the result that when firms face demand cycles, conditional on current demand, prices will be higher if demand is expected to rise in the future, compared to if it is expected to fall. 1 We show that when firms face endogenous capacity constraints, these predicted pricing patterns can change; therefore, ignoring capacity constraints may lead us to conclude collusion does not exist when, in fact, it exists.

Collusion when firms face capacity constraints, has recently been the focus of a number of antitrust cases in both the US and Europe. The US Department of Justice recently launched an investigation into capacity collusion in the DRAM market. Mergers increasing the ability of firms to coordinate on capacity levels have also concerned the European Commission. The commission blocked the Airtours and First Choice Holidays merger in the package holiday travel market partly because of concerns about an increased coordination in capacities; in this industry capacity levels are chosen well in advance of consumer bookings. Coordinating on capacity was also the commission’s initial objection to the UPM–Kymmene/Haindl newsprint merger. 2 Antitrust policy makers have noted the difficulty of uncovering collusion in markets with strict capacity constraints. Coordinating on capacities can lead to lower capacity levels and outputs closer to these capacity levels. Therefore, if one observed the market, taking capacity levels as given, they may conclude that collusion does not exist, presuming instead that firms are simply capacity constrained. This added difficulty increases the importance of understanding how prices behave when firms collude on both capacities and prices.

In this paper, we analyze the collusive behavior of firms where changing the scale of operation takes a significant period of time and market demand fluctuates cyclically. Our goal is to establish testable implications with respect to pricing behavior along the demand cycle. At the beginning of the game, firms choose capacity levels, which are

1 We thank Severin Borenstein, Massimiliano De Santis, Joseph Harrington, Jeroen Hinloopen and Louis Makowski. Seminar participants at UCD and the 2006 International Industrial Organization Conference also provided constructive comments.

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1 See, for example, Borenstein and Shepard (1996) and Rosenbaum and Sukhar-omana (2001).

2 While the commission ultimately concluded that coordinating on capacities was too difficult, our analysis suggests that this conclusion may have been unwarranted.

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doi:10.1016/j.ijindorg.2009.07.009

Please cite this article as: Knittel, C.R., Lepore, J.J., Tacit collusion in the presence of cyclical demand and endogenous capacity levels, Int. J. Ind. Organ. (2009), doi:10.1016/j.ijindorg.2009.07.009
assumed to be fixed for the remaining time periods. Firms then face deterministic demand cycles and compete in prices. This represents industries where the cycles are frequent, or capacity is difficult to alter; for example, the holiday travel market cited above, electricity markets where demand cycles each day and capacity changes can take over 18 months, or the gasoline refining market where demand exhibits annual fluctuations while refining capacity has remained fairly constant.3

We show that the inclusion of the scale decision as a formal choice variable in the dynamic game can drastically change the collusive pricing patterns and the effectiveness of collusion. Capacity constraints have two countervailing effects on firms’ ability to collude. First, low capacity levels may reduce the incentive for a firm to deviate from collusion by limiting the immediate gain from defection. That is, if a firm’s capacity is less than the market demand at the collusive price, the firm cannot supply the entire market after a low price deviation. On the other hand, low capacity levels can decrease the severity of the credible punishment after a deviation, increasing the incentive to defect. We find that whether low or high capacities best facilitate collusion depends on how expensive it is to install; hence, the price of capacity is a key determinant of collusive behavior. Furthermore, we find that endogenizing capacity constraints can have a significant impact on the efficacy of collusion, since firms are able to choose capacity levels to increase the collusive profits.

We build on two papers which have integrated capacity constraints into dynamic games with non-stationary demand: Staiger and Wolak (1992) and Fabra (2006).4 Staiger and Wolak study a repeated game with independently identically distributed demand shocks each period. The capacities are chosen before the demand state is realized, while prices are chosen after. They restrict analysis to exclusively symmetric capacities, simplifying the analysis greatly. They find the collusive pricing path might have periods of both mild and severe price wars. The mild price wars are a joint lowering of symmetric prices below the monopoly level, while the severe price war is a period where firms revert to non-cooperative mixed pricing on the collusive path. Because the severe price wars can be on the equilibrium path, collusive market shares are unstable over time.

Fabra (2006) adds symmetric exogenous capacity constraints to the model of Haltiwanger and Harrington (1991). The focus of the analysis is on showing that Haltiwanger and Harrington results are not robust to this setting. She restricts the analysis to equilibria where prices are arbitrarily close to constrained-monopoly levels. This greatly simplifies the analysis, but also leaves the richness of collusion pricing in this model unexplored. In contrast, in our model firms endogenously choose capacities, which are free to be asymmetric (and often are asymmetric in equilibrium).5 Furthermore, we characterize the whole range collusive equilibria.

Our primary analysis is concerned with collusive pricing patterns, specifically the existence of price wars and pricing during booms and busts. We identify two types of price wars. The first is similar to the pricing patterns in Rotemberg and Saloner (1986) which show that when each period’s demand depends on an independent identically distributed (iid) shock there are no capacity constraints, prices may be inversely correlated with the level of demand. Because the gain from cheating is greater when demand is higher (and the punishment is independent of the current level of demand), prices may fall when demand increases in order to counteract the incentive to cheat; we refer to these counter-cyclical prices as mild price wars.6

A second type of price war also exists. Consistent with Staiger and Wolak (1992), we find under certain capital prices and discount levels, firms will switch between cooperative and non-cooperative mixed-pricing behavior; we refer to these periods as severe price wars. As the marginal cost of capacity increases, severe price wars are only possible in periods of higher and higher demand. Unlike mild price wars, severe price wars correspond to periods where one firm actually undercuts the other in equilibrium.7

Our results provide guidance to policy makers in two ways. First, we establish testable implications with respect to the relative prices during booms and recessions; these can be used to uncover pricing patterns consistent with tacit collusion. We show that when the marginal cost of capacity is high, for equal current demand levels, prices in booms are generally lower than prices in recessions. In this case, capacity is too costly to hold as extra punishment.8 Instead, firms choose low capacity levels to limit the gain from deviations in high demand periods; these low capacity levels lead the near-term punishment after a defection to be smallest when demand is growing. A weaker converse is also true: if the cost of capacity is low, then there is an equilibrium such that prices in the booms are generally higher than prices in the recessions. In this case, capacity is cheap enough that holding large amounts, to increase the credible level of punishment, is most helpful to maximize collusive profits. Therefore, the near-term loss after a deviation is largest when demand is growing. If the cost is in an intermediate range, no such blanket pricing patterns can be established; the relationship can change along the demand cycle. Second, we also show that, for a given discount rate, collusion is more effective the lower the marginal cost of capacity. That is, as the marginal cost of capacity lowers, collusion at the monopoly prices becomes possible for a wider range of discount rates.

To further examine pricing patterns, we calculate numerical examples for different capacity costs at varying discount factors. Numerical examples with extremely low capacity costs show very similar pricing patterns to the limitless capacity model of Haltiwanger and Harrington (1991). As in Haltiwanger and Harrington, we also find that prices are pro-cyclical for high discount factors, but can become extremely counter-cyclical if firms are impatient enough. With high capacity costs, at equal current demand levels, collusive prices are lower in the boom than in the recession and never become counter-cyclical. The prices always remain high in the largest demand periods, while severe price wars can occur in the lowest demand levels.

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3 Other industries such as cement, railroad, steel, heavy electrical equipment and petroleum also loosely fit this abstract description. Scherer and Ross (1990) describe these industries as all having relatively high concentration, high fixed costs, relatively low marginal costs and non-stationary demand patterns. The high fixed costs come from the requirement of long-term pre-commitment to production technologies and/or resource investment.

4 There is a substantial literature on collusion behavior of industries with capacity constraints, price competition and stationary demand; including Brock and Scheinkman (1985), Benoit and Krishna (1987), Davidson and Deneckere (1990), Lambson (1994), Compte et al. (2002) and Dechenaux and Kovenock (2003). Lambson (1994), Compte et al. (2002) and Dechenaux and Kovenock (2003), have analyzed collusion with asymmetric capacities, but assume demand is constant over time.

5 We follow the existing literature and refer to booms as periods where demand is growing and recessions as periods where demand is contracting.

6 While this does not represent a price war in the sense that firms revert to non-cooperative pricing, we keep the nomenclature of Rotemberg and Saloner (1986).

7 Green and Porter (1984) also find reversion to non-cooperative behavior in a model of collusion; however, this is driven by information asymmetries rather than capacity constraints.

8 We follow the existing literature and refer to booms as periods where demand is growing and recessions as periods where demand is contracting.

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### Table 1

<table>
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<th>Discount</th>
<th>Monopoly</th>
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<th>Non-cooperative</th>
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2. Model basics

Consider an industry with two infinity-lived firms producing a homogeneous product with a common discount factor $\delta \in (0,1)$. The index $i$ is used to identify an arbitrary firm, where $i \in \{1,2\}$. Throughout the paper, we use $j$ to index the firm other than $i$. Demand is assumed to follow deterministic cyclical fluctuations over time based on the parameter $\theta$ that repeat every $\tau$ (finite) periods; we label each $\tau$-period cycle as $\Theta_{t} = \{\theta_{1}, \theta_{2}, \ldots, \theta_{\tau}\}$. Formally, $\theta$ follows the deterministic cyclical time path given by,

$$
\theta_{t} = \begin{cases} 
\theta_{1} & \text{if } t \in \{1, \tau + 1, 2\tau + 1, \ldots\} \\
\theta_{2} & \text{if } t \in \{2, \tau + 2, 2\tau + 2, \ldots\} \\
\vdots & \\
\theta_{\tau} & \text{if } t \in \{\tau, 2\tau, 3\tau, \ldots\}.
\end{cases}
$$

The market demand function at any time $t$, given the state $\theta_{t} \in \mathbb{R}_{+}$, is $D(\cdot, \theta_{t}) : \mathbb{R}_{+} \mapsto \mathbb{R}_{+}$. The inverse demand function at any time $t$, given the state $\theta_{t} \in \mathbb{R}_{+}$, is $P(\cdot, \theta_{t}) : \mathbb{R}_{+} \mapsto \mathbb{R}_{+}$.

We will often make use of the two extreme points on the demand curve, for a given $\theta$: the demand at zero prices and the price at which demand is zero. These values are assumed to be finite, $D(0, \theta) < \infty$ and $P(0, \theta) > 0$, for all $\theta \in \mathbb{R}_{+}$, respectively. We also define the maximum of these extreme points across all $\theta$. We denote $D_{\text{max}} = \max_{\theta \in \mathbb{R}_{+}} D(0, \theta)$, the highest demand that can be achieved in a cycle, and $P_{\text{max}} = \max_{\theta \in \mathbb{R}_{+}} P(0, \theta)$, the highest price possible, with a non-zero quantity, in a cycle.

We label a single firm's price at time $t$ as $p_{ij} \in [0, P_{\text{max}}]$ and the pair of prices at time $t$ by $p_{i}$. Each firm has a capacity $x_{i} \in \mathbb{R}_{+}$, the absolute limit on the number of units it can produce. The marginal cost of production is zero up to the firm's capacity and infinite for any quantity beyond. We denote a pair of capacities by $x = (x_{1}, x_{2}) \in \mathbb{R}_{+} \times [0, D_{\text{max}}] \times [0, D_{\text{max}}]$.

The first two assumptions establish the basic properties of the industry demand function.

**Assumption 1.** For each $\theta \in \mathbb{R}_{+}$, the quantity $D(0, \theta)$ is such that for all $q \in [D(0, \theta), P(q, \theta) \in (0, \infty)$ and for all $q \geq D(0, \theta)$, $P(q, \theta) = 0$. On $(0, D(0, 0), 0)$, $P(q, \theta)$ is twice-continuously differentiable, strictly decreasing and concave in $q$.

**Assumption 2.** Demand is increasing in $\theta \in \mathbb{R}_{+}$, such that for all $P \in (0, P(0, \theta))$, $D(P, \theta) > D(P, \theta')$ if and only if $\theta' > \theta$.

The next two assumptions specify the properties of the capacity cost function each firm faces.

**Assumption 3.** The capacity cost function is homogeneous across firms. The marginal cost is the constant $c > 0$, such that each firm pays a per-period cost $c_{i}(x_{i}) \equiv c_{x_{i}}$ for $i \in \{1,2\}$.

**Assumption 4.** The industry cost of capacity permits positive profit:

$$c < \tau(i) = \frac{\sum_{t=1}^{\tau} \theta_{t}}{\sum_{t=1}^{\tau} \theta_{t}^{2}}.$$

Our final assumption is important; this minimal regularity of the demand cycles is required to make clear statements describing colusive pricing and revenue patterns.

**Assumption 5.** The cycle has a single peak, i.e., there exits $\tilde{\theta} \in \{1, \ldots, \tau\}$ such that

$$\theta_{1} < \theta_{2} < \ldots < \theta_{\tilde{\theta}} > \ldots > \theta_{\tau - 1} > \theta_{\tau}.$$

From this point onward we impose Assumptions 1–5, although only the results pertaining to pricing patterns over the demand cycle (Theorem 1–3) require Assumption 5.

Two important features of this model are the deterministic nature of demand cycles and the exogenous firm structure. As such, this model pertains to industries where entry or exit of firms is unlikely to happen.\(^{11}\)

The timing of the game:

**Period 0:** At the beginning of the game, each firm $i \in \{1,2\}$ chooses $x_{i}$ independently and simultaneously; capacities remain at these levels throughout the game.

**Period 1:** Each firm $i$ observes the other firms capacity realization $x_{j}$ and chooses $p_{i1}$ independently and simultaneously.

**Period 2:** Each firm observes the other firm’s choice $p_{j1}$. Then the firms choose $p_{i2}$, independently and simultaneously.

**Period $t$:** Each firm observes the others firm’s choice $p_{jt-1}$. The firms choose $p_{jt}$ independently and simultaneously.

2.1. The non-cooperative equilibrium

We begin by discussing the non-cooperative equilibrium. Because each demand cycle of pricing games is the same, the subgame perfect equilibria for a single cycle will be all that is needed to construct the non-cooperative subgame perfect equilibria of the infinitely repeated game.

2.1.1. The pricing stage-games

There is a pricing stage-game for each $\theta \in \Theta$, that follows the initial capacity choice. Here we fix the two firms’ capacities at arbitrary

\(^{9}\) Similar assumptions about the structure of demand were introduced in Haltiwanger and Harrington (1991), although the assumptions we use are more similar to Fabra (2006).

\(^{10}\) The cost of capacity $c_{x}$ is paid each pricing period, thus at the time of the capacity decision the discounted future cost of capacity is $\sum_{t=1}^{\tau} \theta_{t} c_{x}$. This specification follows Benoît and Krishna (1987), although there would be only minor notational changes if we instead defined costs as a one time payment.

\(^{11}\) The deterministic structure of demand has proven itself useful in the case of the Bertrand supergame and provided results conceptually analogous to the uncertain Markov setting where the end of booms and recessions is unknown. Here we are referring to the similarity of results in Haltiwanger and Harrington (1991) with deterministic demand cycles relative to Bagwell and Staiger (1997) with Markov demand cycles.
values \( x_1 \) and \( x_2 \) and examine the pricing for each demand level \( t \). We assume demand is rationed using the surplus maximizing or efficient rationing rule. Under our assumptions Dasgupta and Maskin (1986a, b) have shown the existence of the Nash equilibrium in each pricing stage-game. Following Kreps and Scheinkman (1983) and Deenecke and Kovenock (1992) the revenue of the pricing subgames can be split into three regions. To help characterize the equilibrium revenue, we define the Cournot best response function at a zero cost of capacity,

\[
r(x_t, \theta_t) = \arg \max_{q_t \in [0, 0.5] \cdot x_t} P(q + x_t, \theta_t). \]

The unique-equilibrium expected revenue function for firm \( i \) given capacities \( x_1 \) and \( x_2 \) is,

\[
R^i_t(x_t, \theta_t) = \begin{cases} 
P(x_t + x_2, \theta_t) x_t & \text{if } x_t \leq (x_2, \theta_t) \text{ and } x_2 \leq (x_1, \theta_t) \\
0 & \text{if } \min[x_t, x_2] > D_t(0, \theta_t) \\
P(r(x_t, \theta_t) + x_i, \theta_t) r(x_t, \theta_t) & \text{otherwise, and } x_i > x_t \\
p_i(x_t, \theta_t) x_t & \text{otherwise, and } x_t > x_i 
\end{cases}
\]

The last two regions only have mixed-strategy equilibrium pricing. The Nash equilibrium expected revenue for the smaller firm is determined by the lowest price in the mixed-strategy support. This is given as,

\[
p_i(x_t) = \max \left\{ \frac{P(r(x_t, \theta_t) + x_i, \theta_t) r(x_t, \theta_t)}{x_i}, \min_{p_t \in R_t} \{ p_t \mid P_t(x_t, \theta_t) = P(r(x_t, \theta_t) + x_i, \theta_t) r(x_t, \theta_t) \} \right\}
\]

We highlight that \( R^i_t(x_t, \theta_t) \) is continuous in \( x_t \) on \( \mathbb{R}^2 \). We denote the Nash equilibrium pricing of the period \( t \) stage-game, given capacities \( x_1 \) and \( x_2 \), as \( p^*(x_t) = p^*(x_t, \theta_t) \). The Nash equilibrium pricing of each stage-game constructs the non-cooperative subgame perfect equilibrium of the entire pricing cycle game. The reasoning is as follows: if in each time, \( t \geq 1 \), the other firm prices according to the Nash equilibrium of each individual stage-game, the best response is to price Nash in the current and all subsequent stage-games. For notational simplicity, we define \( R^i_t(x_t) = R^i_t(x_t, \theta_t) \).

2.1.2. The capacity choice stage-game

Kreps and Scheinkman (1983) prove that in a sequential capacity and price game with one period of pricing, the unique Nash equilibrium is in pure strategies and has the same price and quantity sold as the analogous Cournot game. In our setting this is not always the case. This is because the demand level fluctuates over a single capacity choice. Indeed, even the existence of an equilibrium with symmetric pure capacities is not guaranteed. However, using the fact that each firm’s expected profit function is continuous in the capacity choices, existence of a mixed capacity equilibrium is guaranteed based on Glicksberg (1952). We denote the set of equilibria by \( X^N \) and each equilibrium by \( \tilde{R}^i_t \).

2.2. Additional notation for collusion equilibria

We require additional notation to discuss the dynamics of prices, revenues and profits within a collusive equilibrium. We define \( P_t \) as the set of symmetric prices in \( [0, \bar{P}_{\text{max}}] \times [0, \bar{P}_{\text{max}}] \). Given capacity choices, the relevant price space in the dynamic game is determined by the capacities and is given as \( P_t(x_t) = P_t \cup \bar{P}_t(x_t) \) for period \( t \).

Define a pricing path \( p_t = (p_t)_{t=1}^\infty \in P_t(x_t) = \Pi_t^n P_t(x_t) \) and continuation path from time \( t \) onward, \( p_t = (p_t)_{t=1}^\infty \).

Firm \( i \)'s period \( t \) revenue is,

\[
R_i(p_t, x_t) = \begin{cases} 
\rho_t \min\{x_t, D(p_t, \theta_t)/2\} & \text{if } p_t \in P_t, \\
\rho_t(x_t) & \text{if } p_t \in \bar{P}_t(x_t). \end{cases}
\]

At times we will refer to the sum of revenues across both firms within a given time period and the present discounted value of a firm’s revenues; we denote the joint revenue at time \( t \) by \( R_j(p_t, x_t) = R_t[p_t(x_t) + R_t(p_t, x_t)] \), and the total discounted revenue of firm \( i \) by \( R_t[p_t(p_t, x_t, \theta_t) = \sum_{t=1}^\infty \rho_t(p_t(x_t) \theta_t) \). Similarly, the discounted profit of firm \( i \) is \( \Pi_t(p_t, x_t, \theta_t) \). Denote by \( \Pi_t(p_t, x_t, \theta_t) \), the firm’s joint discounted profit. The profit of firm \( i \) at capacities \( x_t \) with non-cooperative prices is \( \Pi_t^N(x_t, \theta_t) \).

3. Basics of the collusive equilibria

In this section, we establish some basic definitions and results which are the foundation of the analysis of collusive behavior in the infinite-time game.

3.1. Joint profit maximizing prices and capacities

At the upper bound, when the firms are extremely patient (or, analogously, the period length is very short), the collusive equilibrium might be the joint profit maximizing prices and capacities. As a building block for the joint profit maximizing prices and capacities we study the monopoly solution. It may also be the case that firms can sustain monopoly prices, conditional on their capacity choices, but are not patient enough to install the monopoly capacity level. Given the presence of capacity constraints, we distinguish between unconstrained-monopoly prices, \( p_{t}^m \), and constrained-monopoly prices, \( p_{t}^{cm}(X) = \max[\Pi_{t}^{cm}(X, \theta_t)] \) for any \( t \). The monopoly capacity choice is labeled \( X^m \) and the constrained-monopoly price at the monopoly capacity \( P_{t}^{cm} = P_{t}^{cm}(X^m) \) for all \( t \).

In any period \( t \), constrained-monopoly pricing is either unconstrained-monopoly pricing or higher pricing such that demand equals the capacity constraint. If the two firms have symmetric capacities it is fairly obvious that constrained-monopoly prices will maximize joint profit. In contrast, if the firms are asymmetric the pricing that maximizes joint profit is not immediately clear. Denote by \( \bar{P}_t^m(x_t) = (p_{t}^m(x_t + x_2), p_{t}^m(x_t + x_1)) \), the symmetric pair of contained monopoly prices at capacities \( x_t \).

**Proposition 1.** For all \( \delta \in (0, 1) \) and \( \epsilon \in (0, \epsilon(\delta)) \),

1. There exists a unique monopoly solution \( X^m_\epsilon \) and \( P_{t}^{cm} \).
2. At capacities \( x_t \) the joint profit maximizing prices are constrained-monopoly prices \( p_{t}^{cm}(X) \).
3. All joint profit maximizing capacities are such that \( x_1 + x_2 = X^m_\epsilon \).

---

\[12\] The prices \( p^*_i(x_t) \) are expressed as a measure over the space \( [0, \bar{P}_{\text{max}}] \times [0, \bar{P}_{\text{max}}] \) to accommodate for mixed strategy equilibrium.

\[13\] Reynolds and Wilson (2000) show the non-robustness of the result of Kreps and Scheinkman (1983) to demand uncertainty for the case of continuously distributed uncertainty. The non-cooperative equilibria of our game are analogous to a game with discretely distributed uncertain demand. Lepore (2008) characterizes the set of non-cooperative equilibria of a game of this form. It is shown that there are equilibria that involve pure symmetric capacities in only two cases: (i) the demand cycle has very little variance in demand periods and the cost of capacity is relatively high, or (ii) the demand cycle includes many large similar demand periods, few very small demand periods and the cost of capacity is relatively low. For any demand cycle that does not fit the description of (i) or (ii), all non-cooperative equilibrium will involve asymmetric or mixed strategy capacities.

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Please cite this article as: Knittel, C.R., Lepore, J.J., Tacit collusion in the presence of cyclical demand and endogenous capacity levels, Int. J. Ind. Organ. (2009), doi:10.1016/j.iindorg.2009.07.009
3.2. The incentive constraints for collusive for pricing

All subgame perfect equilibria supported by trigger strategies can be characterized by pricing and capacity incentive compatibility constraints. The set of incentive compatible prices at each period must be such that the gain from a deviation in price cannot exceed the anticipated loss from punishment in the future, which is the difference between the discounted future profit on the collusive equilibrium path and discounted future profit of the punishment path.

Suppose no firm deviated in the capacity stage, capacities (possibly asymmetric) are fixed and the collusive phase continues. If a firm has deviated from collusive pricing, then they expect the other firm to retaliate by playing the infinite reversion to the non-cooperative equilibrium at the fixed capacities. The use of non-cooperative reversion, instead of an optimal penal code, is innocuous to the pricing game.16

3.2.1. The incentive compatibility constraints and prices

The arbitrary pricing strategy \( p \) is in the set of incentive compatible collusive prices if it is such that,

\[
\mu_{it}(p_t, x) \leq \lambda_{it}(p_{t+1}, x, \delta), \quad \text{for all } i \in \{1, 2\}, \text{ and } t \in \{1, 2, \ldots\}
\]

where,

\[
\mu_{it}(p_t, x) = \begin{cases} \sup_{\nu \in [0, \nu_{\max}]} \max_{j \neq i} (p_{ij}(x) - R_{it}(p_t, x)) & \text{if } p \in P_i, \\ 0 & \text{if } p \in P_N, \end{cases}
\]

\[
\lambda_{it}(p, x) = \sum_{s=0}^{\infty} \delta^{s-1} (R_{is}(p_t, x) - R_{is}(x)).
\]

Define the set of prices that satisfy the constraint set at a given capacity and discount factor as \( \Delta(x, \delta) \). Formally, for all \( x \in X \) and \( \delta \in (0, 1) \),

\[
\Delta(x, \delta) = \left\{ p \in P(x) \mid \mu_{it}(p_t, x) \leq \lambda_{it}(p_{t+1}, x, \delta), \right\}, \quad \forall i \in \{1, 2\} \text{ and } t \in \{1, 2, \ldots\}.
\]

3.2.2. Under-cutting and over-cutting

The pricing stage-game incentive compatibility constraints can naturally be separated into two constraints. The first is the standard collusive pricing constraint for firms under-cutting the collusive price, guaranteeing that the immediate gain from an undercut cannot exceed the future loss from punishment. The second is a by-product of capacity constraints that only applies if the under-cutting constraint is already binding. If the price in any period is bound significantly below the monopoly level by the under-cutting constraint, then the possibility of a higher price deviation also exists. This over-cutting constraint is a feature unique to models with capacity constraints since, depending on the rationing rule, a firm may have an incentive to increase its price.

3.3. The incentive constraints for capacities

Capacity choices also involve incentive compatibility constraints to insure no deviation from the collusive capacity level is beneficial to either firm. If neither firm defects in the capacity stage, then they continue into collusive pricing. If a firm deviates from collusive capacities, then they expect the other firm to retaliate by playing the infinite reversion to the non-cooperative equilibrium prices at the previously chosen capacity levels.

3.3.1. The incentive compatibility constraints and capacities

An arbitrary capacity is in the set of incentive compatible collusive capacities, given prices \( p \), marginal cost of capacity \( c \), and discount factor \( \delta \), if \( x \) is in the set \( \Phi(p, c, \delta) \) as defined in (5).

\[
\Phi(p, c, \delta) = \left\{ x \in X \mid \Pi_i((p, x, c, \delta) \geq \max_{z \in [0, \nu_{\max}]} \rho^N_i(z, x, y) \right\} \forall i \in \{1, 2\},
\]

4. Collusive equilibria

In this section, we restrict firms to collude by way of pure capacity strategies and symmetric pricing strategies which lead to the highest discounted joint profit.18

Much of the intuition from this generalization remains if we focus on pure strategies. Our restrictions imply that we focus on the “most-collusive equilibrium.” Most-collusive prices are the joint profit maximizing subgame perfect prices in the set of pure symmetric and non-cooperative prices for any fixed capacity pair \( x \); we denote most-collusive prices as \( p^c(x) = (p^c_i(x))_{i \in \{1, 2\}}. A \text{ most-collusive equilibrium (MCE) is a joint profit maximizing subgame perfect equilibrium with most-collusive prices and capacities leading to the highest discounted joint profit.}

4.1. Existence of collusive equilibria

In this section, we state the existence results for most-collusive equilibria.

Proposition 2. For all \( \delta \in (0, 1) \) and \( x \in X \), there exists a most-collusive pricing solution \( p^c(x) \) such that,

\[
p^c(x) = \arg \max_{p \in X(c, \delta)} \left\{ R^c(p, x, \delta) \mid p \in \Delta(x, \delta) \right\}.
\]

We define the set of incentive compatible capacities at most-collusive prices as \( \Phi(p^c, c, \delta) \). Any capacities \( x^c \in \Phi(p^c, c, \delta) \) are collusive equilibrium capacities. There are many model specifications such that the set of collusive capacities is large, often including many equilibria with high levels of capacity asymmetry.

The MCE capacities, \( x^{MC} \), are the joint profit maximizing capacities in \( \Phi(p^c, c, \delta) \). If the set of \( \Phi(p^c, c, \delta) \) is empty, then non-cooperative capacities are the only possible pure capacity subgame perfect equilibria.

15 Following the nomenclature in Abreu (1986, 1988), reversion to non-cooperative pricing will support the same set of prices as the optimal penal code because, for symmetric prices, the weakly larger capacity firm always has the tighter binding constraint and its non-cooperative profits are at min-max levels.

16 In the case of exogenous capacity constraints, Lambson (1987, 1994) proves that an optimal punishment exists and is at the min-max expected revenue level for the deviating firm with most capacity pair. For these capacities, reversion to non-cooperative pricing forever yields the min-max payoffs. There is only one case that non-cooperative reversion does not lead to the same payoffs as min-max reversion. This when the defector is the smaller firm and for some periods on the cycle its non-cooperative revenue is not dependent on the larger firms capacity.

17 With regards to capacity deviations, the use of non-cooperative reversion is not necessarily innocuous. Particularly, if a firm defects to a lower capacity than its rival, then it might not receive the optimal punishment. This affects the span of the set of possible subgame perfect capacities. However, this assumption does not affect the abstract statements of Theorems 1 and 2.

18 Knittel and Lepore (2006) considers mixed strategy capacities and restricts firms to pricing strategies, on the equilibrium path, with per-period firm level revenue no lower than the non-cooperative expected revenue. Under these assumption, existence of MCE is generic. The general setting of mixed strategies guarantees that the constraint set is non-empty, while the restriction on payoffs along the equilibrium path insures the constraint set is compact and the maximand is upper semi-continuous.
4.2. Key discounts

There are two key discount factors which characterize the most-collusive equilibria. If the discount factor is close enough to one, joint profit maximizing prices are sustainable at all collusive equilibrium capacities.

**Proposition 3.** There exists \( \delta(c) \in (0,1) \) such that for all \( \delta \in [\delta(c),1] \) and \( x^t \in \Phi(p^c, c, \delta) \), \( P_{x^t}^c = p^m(x^t) \).

The proof of Proposition 3 is based on first establishing that there is a discount factor where all collusive prices are at the joint profit maximizing level. This is akin to a Folk theorem for the capacity constrained pricing game.\(^1\) To show this for a single period \( t \), we fix capacities at an arbitrary level and all other prices at the joint profit maximizing levels. The joint profit maximizing price level is sustainable in period \( t \), if the discount factor is one. The fact that the future loss is continuous in \( \delta \) and goes to zero as \( \delta \) goes to zero, is used to prove that the joint profit maximizing prices are sustainable if and only if the discount is greater than or equal to \( \delta^*(x) < 1 \). The discount factor \( \delta^*(c) \) is the smallest discount such that all MCE are at joint profit maximizing levels for all capacity levels.

In contrast, there is a largest discount factor such that for any discount factor below this level the only equilibrium path is non-cooperative capacities and prices.

**Proposition 4.** There exists \( \delta(c) \in (0,1) \) such that for all \( \delta \in (0,\delta(c)) \) the only equilibria involve non-cooperative capacities and prices.

In all that follows, the range of discount factors where the most-collusive equilibrium is primarily studied, lie between \( \delta(c) \) and \( \delta^*(c) \). In our numerical examples, the upper bound discount \( \delta^*(c) \) is approximately 0.64 in the lowest cost example, and approximately 0.94 when costs are extremely high. The lower bound discount \( \delta(c) \) ranges from just above 0.5 at the lowest cost, to approximately 0.76 when costs are high. Both discount factors tend to increase as the cost of capacity increases so that the interval of discount factors between \( \delta(c) \) and \( \delta^*(c) \) has a fairly wide range across different levels of \( c \).

These two propositions construct the outline of the most-collusive pricing picture shown in Fig. 1. We will expand on this diagram throughout the section to complete the picture of the most-collusive pricing behavior.

4.3. Basic character of collusive pricing

We focus on describing the character of most-collusive prices for discounts in the range \( (\delta(c), \delta^*(c)) \) where all prices are neither always joint profit maximizing or always non-cooperative. Define the set of prices which are incentive compatible at time \( t \) given most-collusive equilibrium pricing from \( t+1 \) on

\[
\Delta_t^c(x, \delta) = \left\{ p_1 \in P_t(x), \begin{array}{l}
\xi_{t,t+1}(p_1, x) \leq \xi_{t+1}(p_{t+1}, x, \delta) \\
\text{and } R_i(p_1, x) > R_i(p_{t+1}^c, x)
\end{array} \right\}.
\]

The set \( \Delta_t^c(x, \delta) \) is the prices that are incentive compatible, pure symmetric, and result in higher joint expected revenue for period \( t \) than non-cooperative pricing.

Based on Proposition 5 there are three basic most-collusive pricing patterns. Which of the three pricing patterns a period follows depends on the discount factor and how the demand parameter of a given time period relates to the capacities.

Region 1 (Joint profit maximizing prices): the collusive price for a period of demand in this region is sustainable at or above the single-period joint profit maximizing level. If the capacity constraint binds so strongly that no undercut can increase the demand for both firms, then the most-collusive price is above the single-period unconstrained-monopoly level. This is where \( \theta_t \) is such that \( D(P_t^{um}, \theta_t) \geq x_1 + x_2 \). Pricing is higher than the single-period unconstrained-monopoly level for time period \( t \).

In contrast, the prices are at or below the unconstrained-monopoly level if \( D(P_t^{um}, \theta_t) \geq x_1 + x_2 \), thereby permitting sustainable joint surplus maximizing revenue.

Region 2 (Mild price wars): these are periods such that each firm’s under-cutting incentive compatibility constraint binds at the joint maximal prices, but the over-cutting constraint does not play a role, i.e. \( \Delta_t^c(x, \delta) \neq \emptyset \). In this region, \( p_t^c \) is lower than \( P_t^{um}(x_1 + x_2) \) and is the highest price that is under-cutting incentive compatible. More precisely, \( p_t^c \) is the largest symmetric price such that the incentive constraint holds with equality for period \( t \).

Region 3 (Non-cooperative pricing): in this region, non-cooperative prices yield the highest expected revenue of any prices that satisfy the incentive compatibility constraints. Thus, the pricing and revenue will be at the non-cooperative levels.

A severe mixed price war period must be within this region. In particular, mixed price wars occur for demand periods where, for at least one firm \( i \), \( x_i \in (r(\theta_t), D(\theta_t)) \) and for all \( p_{1,t} = p_{2,t} = P_t \) that are incentive compatible, \( P_t D_i(P_t, \theta_t) \leq R_t^c(x) \).

4.4. Basic character of the MCE capacities

In this section we focus on a characterization of MCE capacities. The monopoly capacity choice given most-collusive prices is

\[
X^* = \arg \max_{X \in [0, X_{max}]} 2 \sum_{1}^{\infty} \delta \left( R_j(p_t^c(X, X)) - cX \right).
\]
Label $\chi^t$, the smallest capacity in the set of maximizers of (8). Based on the definition of a $\chi^t$, we characterize the set of MCE capacities.

Proposition 6. For all $\delta \in (\hat{c}(c), \hat{c}(c))$ and $c \in (0, \hat{c}(c))$, if there exists $x \in \partial(p^*, c, \delta)$, then $x^{mc}$ is symmetric and $x^{mc} \geq X^*/2$.

Unlike the price choices, there is no explicit symmetry assumed in MCE capacities. In spite of this, if there are any incentive compatible capacities preferred to the non-cooperative capacities, then the MCE capacities will always be pure and symmetric. The strong tendency towards capacity symmetry is a consequence of symmetric pricing that permits firms to make the highest joint revenue when they are symmetric. The proof of the proposition is based on the following lemma.

Lemma 1. The incentive constraint of the (weakly) larger capacity firm is the constraint that binds most-collusive pricing.

This lemma is the basis for symmetric most-collusive capacities. Suppose we take a fixed capacity level and distribute the capacities of the two firms unequally. If the firms become more similar in size, then the gains of the larger firm are weakly less from a defection and its losses from future punishment will also become greater. This creates additional slack in the incentive constraint for the larger firm. Since it is only the larger firms’ constraint that binds under most-collusive pricing, the movement towards symmetry weakly increases prices in all periods.

Based on Proposition 6 there is the following basic trichotomy of MCE capacities, depending on the discount factor and marginal cost of capacity:

Region 1 (joint maximal capacities): if the capacity incentive constraint does not bind at the joint profit maximizing capacities, then $x^{mc} = (X^*/2, X^*/2)$, such that $X^*$ is a solution to the joint maximal program (8). This is half the capacity a monopolist would choose given most-collusive pricing. The most-collusive prices might be different than the constrained-monopoly prices. This difference is likely to lead to a different choice of joint profit maximizing capacities. Hence, the MCE capacities are not necessarily half the monopolist’s capacity.

Region 2 (symmetric non-joint maximal capacities): in this region, the capacity incentive constraints bind such that any of the joint maximal solutions defined above are not incentive compatible, but there is some incentive compatible capacities that are preferred to the non-cooperative capacity choices. In this case, the capacities will be pure and symmetric. The only incentive compatible capacities are larger than $X^*/2$. Hence, the MCE capacities can range greatly, with each firm’s capacity lying anywhere between $X^*/2$ and $D_{\text{max}}$. Much of the variation in the size of the capacities will depend on the marginal cost of capacity.

Region 3 (non-cooperative capacities): in this region, the incentive constraint binds so harshly that the only admissible capacities are the non-cooperative capacities. There is no collusion in capacities; instead they are the result of competition with most-collusive prices.

4.5. Pricing patterns

The goal of this section is to understand the pricing patterns of the most-collusive equilibrium when monopoly prices are not sustainable in all periods. Towards this end, we focus on the general properties of prices in booms versus recessions.

The presence of capacity constraints alters both the gains and losses from defecting at constrained-monopoly prices. When demand is sufficiently high, a defecting firm is unable to capture the entire market. Similarly, during high near-term demand periods, the severity of the punishment is reduced since non-cooperative prices are no longer zero. The relative magnitude of these two countervailing incentives drives the pricing patterns we would expect to see under collusion. To establish a concrete example of the effect of changes in demand levels on incentives of the firms, we graph the single-period expected gains and losses from cheating at the monopoly price. The figures illustrate the single-period gains and losses from optimal defection at unconstrained-monopoly prices when demand is of the form, $D(p, \theta) = \theta - p$.

In any period with demand parameter $\theta$, the profit from deviating is determined by its relationship to capacity. There are three distinct forms of the gains relative to $\theta$, depending on the fixed symmetric capacities. For low demand states, the gains from defection rise with demand as the firm is able to undercut its competitor and capture the entire market. There is a point, however, where the firm is not able to meet all of the additional demand from defection and the gains from cheating fall with $\theta$. At some point, an individual firm is producing at capacity under collusion and there are no gains from cheating.

The losses in the first period following defection can also be characterized by the period's value of $\hat{\theta}$ relative to capacity. As with the gains, these losses initially rise and then begin to fall with $\hat{\theta}$. When the capacity levels are sufficiently large, such that prices fall to zero, defection implies a loss of half of the monopoly profits. If the firms are unable to commit to zero prices upon defection, then the losses from defection are reduced by an amount that depends on the installed capacity. Finally, if demand is sufficiently large such that the combined capacity base cannot meet demand at the monopoly price, then there is no penalty from defecting. In Fig. 3 we plot the first period’s losses with demand parameter $\theta$, after a deviation, when $x = 2$.

What will drive our results with respect to pricing along the cycle is that for any two time periods on opposite sides of the cycle $s$ and $t$, such that $\theta_s = \theta_t$, the gains from defection are the same, but the discounted losses are different since the sequence of demand states that follow are not identical. For equal demand levels, whether prices are higher in the boom or the recession, will depend on where

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20 Our two theorems in the next section apply to any two periods in the boom and recession with equal demand, regardless of how asymmetric the demand cycles. The single peaked demand cycle is the key cycle regularity assumption which permits us to prove our results.
non-cooperatively or other scenarios where marginal costs where a consistent inequality between prices in booms and recessions cannot be established. However, when capacity costs are high enough, then the firms will always choose to collude with capacities that are small relative to the demand cycle. At small enough capacity levels, the punishment incurred after a defection will be lowest in the highest demand periods. Now take the two demand periods on either side of the peak. At any given price, the punishment from defecting in the period that precedes the peak will be lower than the losses in the period following the peak. Since, at any fixed price, the gain from a deviation in these two periods is the same, the incentive compatibility constraint in the boom period binds more strongly than in the recession. Hence, both higher revenue and higher prices are sustainable in the comparable recession period.

In a similar vein, consistent pricing patterns can also exist in industries with low marginal cost of capacity. Pricing properties in this case are slightly more delicate than the high cost case. This is because, there is not always a cost of capacity low enough to yield equilibria with consistent pricing pattern found in the model of Haltiwanger and Harrington (1991) for the full discount range. Indeed, Theorem 2 implies the existence of a single collusive equilibrium that follows the pricing pattern. This is a far weaker result than Theorem 1, which applies to all collusive equilibria. It is possible that the MCE never follow this pattern.

Informally, Theorem 1 states that, if the marginal cost of capacity is large enough, then for all \( c > c_0 \) and \( \delta \in (\delta(c), \hat{\delta}(c)) \) if \( \theta_t = \theta^*_t \) where \( 1 \leq t < t' \leq \tau \), then \( 1-4 \). For all \( x^c \in \mathcal{X}(p^*, c, \theta) \) and \( i \in \{1,2\} \)

1. \( R_{ct}(p^*_t, x^c) \leq R_{ct'}(p^*_t, x^c) \)
2. \( R_{ct}(p^*_t, x^c) > R_{ct'}(p^*_t, x^c) \), if \( p^*_c(x^c) \neq p^*_b(x^c) \) and \( p^*_b(x^c) = p^*_0(x^c) \)
3. \( p^*_c(x^c) > p^*_b(x^c) \), if \( p^*_c(x^c) \neq p^*_b(x^c) \) and \( p^*_b(x^c) = p^*_0(x^c) \)
4. \( p^*_c(x^c) = p^*_b(x^c) \), then \( p^*_c(x^c) = p^*_0(x^c) \).

Informally, Theorem 2 states that if marginal cost is small enough, then for \( \leq 1 \) collusive equilibria the following are true, for either firm, with regards to any two periods of equal demand on single cycle: (1) the period in the boom has weakly more expected revenue, (2) if the recession period prices are not joint profit maximizing and the boom period prices are pure strategy, then the expected revenue inequality is strict, (3) if the recession period prices are not joint profit maximizing and pure strategy and the boom period prices are pure strategy, then the boom period prices are strictly more than the recession period prices, and (4) if the boom period pricing is non-cooperative then so are the recession prices.

The intuition behind Theorem 2 is straightforward: if the most-collusive capacity is large enough, relative to the demand cycle, both the immediate gain from a defection and the individual period loss after a defection are increasing in the demand parameter. Therefore,
the future loss from deviating is greater in comparable boom periods than recession periods. At equal current demand levels, the gain from a defection is the same for these two periods. Hence, the incentive constraint binds first and more strongly in the recession period. Both the expected revenues and the prices are higher in comparable boom periods. If cost of capacity is low enough these large capacities will be an equilibrium for a positive mass of discounts below $\hat{\delta}(c)$. This does not imply that collusion is most profitable at these capacities, i.e., there does not always exist a low enough cost such that the MCE will follow the pattern of Theorem 2.

Fig. 4 summarizes the implications of Theorems 1 and 2. In both the constrained-monopoly (upper) and non-cooperative (lower) pricing regions of the figure prices are symmetric over the cycle. In other words, if two periods have equal demand, then they will have equal pricing strategies on the equilibrium pricing path.

The complexity of the model places a limit on the character of MCE pricing we can prove analytically. In particular, the precise range in terms of marginal costs and discount factors for each of the three MCE patterns is left open. In order to establish stronger patterns, we calculate numerical examples under various cost specifications.

5. Numerical examples

5.1. Prices

To provide concrete examples of how capacity constraints affect market equilibria, we parameterize the model and conduct numerical simulations of the most-collusive equilibrium. These numerical examples give a more detailed picture of the collusive pricing patterns over the demand cycle. In doing so, we adopt the functional form assumptions used by Haltiwanger and Harrington (1991), so that the results of our model can be easily compared to those described in their paper. Specifically, demand is parameterized as: $D(p,t) = \theta_t - 400p$, and we use an eight-period cycle which varies in terms of the intercept $\theta_t$ given as $\theta = (100,125,150,175,200,175,150,125)$. We analyze equilibria under three capacity marginal cost levels. For each marginal cost, the discount factor is varied within the range $[\hat{\delta}(c), \delta(c)]$. We focus our attention on (a) the relative pricing during booms and busts, (b) the cyclicity of prices, and (c) the benefits from endogenizing capacity choices.

Fig. 5 is the low cost example; prices follow the predictions of Theorem 2; at the same demand level prices are lower in the recession than in the boom for all discounts plotted. This result is driven by the fact that capacity is so cheap that it is most productive for the cartel to hold excess capacity to increase the severity of punishment. These results mirror those in Haltiwanger and Harrington (1991) where firms are without capacity constraints. The high capacity levels imply that punishment is greatest during high demand periods, reducing the incentive to deviate during booms, relative to recessions. This pricing pattern holds for almost all discount factors, only very close to $\hat{\delta}(c)$ is there a difference. Just as in the limitless capacity setting, prices are strongly pro-cyclical at high discounts, but become counter-cyclical for low enough discount factors. The counter-cyclical stems from the fact that as the discount factor falls, firms reduce prices in the highest demand periods to sustain collusion because the largest one-shot gains are in these periods.

Fig. 6 plots the most-collusive equilibrium prices for a high cost example. The pricing pattern is starkly different than the low cost case; the prices follow the predictions of Theorem 1 and are always strongly pro-cyclical. When the price of capacity is high, equilibrium capacity levels are low. This increases the incentive to deviate during boom periods, relative to an equal demand period in the recession. To counteract this, firms lower prices during booms, relative to an equal demand period in the recession. Despite this, prices never become counter-cyclical because capacity is too expensive to hold for punishment. Instead, the firms keep capacities small to lock in the high profits from the highest demand periods.

Another interesting feature of the high cost equilibrium is that in period 1, at the discount factor 0.8, the collusive pricing is the same as the non-cooperative mixed-strategy pricing given the equilibrium capacities. This is an example of the mixed-strategy price wars detailed in Proposition 5. In this demand period, the two firms will almost surely name different prices in equilibrium; this would have
the appearance of a single-period under-cutting price war that occurs at the beginning of every demand cycle. In the figure, the line from prices 0.05625 to 0.075 represents the continuous support of the mixed-strategy pricing. This severe price war does not occur in the equilibrium for the lower discount factors 0.78 and 0.76. The firms instead find it optimal to choose larger capacities because their additional impatience significantly lowers sustainable price levels at low capacity levels.

Fig. 7 plots the medium cost case. In this example, both pricing patterns of Theorems 1 and 2 are evident. For the two largest discount factors, the prices follow the pattern of Theorem 1; the three lowest discount factors show pricing patterns consistent with Theorem 2. The collusive pricing at the intermediate discount factor 0.68 does not follow either theorem. Instead, we see that prices are lower in the boom for the higher demand levels and lower in the recession for the demand level 125.

These numerical examples underline the importance of including capacity as a strategic tool when analyzing and testing for collusion. Previous empirical papers test for collusion using the Haltiwanger and Harrington (1991) results that, conditional on current demand, prices will be higher if demand is expected to grow. Our results suggest that this may not be a powerful test since collusion may exist even if prices do not follow the predictions of Haltiwanger and Harrington. Furthermore, the medium cost case implies that these inequalities may change along the demand cycle. This suggests that empirical tests may want to focus on periods around exogenous changes in the cost of capacity and explicitly test for an inequality reversal.

5.2. Profits

By endogenizing capacity, firms are able to cater capacity choices to best facilitate collusion. Next, we compare the benefits of capacity as a strategic tool by calculating equilibrium profits under four scenarios: monopoly, colluding in both capacity and prices, colluding in prices but having non-cooperative capacities, and non-cooperative behavior (Tables 1–3). The third scenario represents markets where collusion takes place after capacity choices have been made; Davidson and Deneckere (1990) refer to this as semi-collusion. We calculate profits, relative to monopoly levels for each of the discount/capacity cost combinations discussed above. The results are striking.

While it is not surprising that there are large differences between the non-cooperative profits and the other three scenarios, we find that including capacity as a strategic tool can have large effects on profits levels, especially when capacity costs are low. As the discount factor in the low cost scenario drops below 0.56, semi-collusive profits fall dramatically, while profits when firms collude both in prices and capacities remain near monopoly levels.

6. Conclusion

We establish a predictive theory of collusive pricing over demand cycles for homogeneous product industries with endogenous long-run capacity and short run price competition. Two key features drive the results in our model: (i) because of the capacity constraints, gains from deviating from collusive prices do not increase monotonically with demand; and (ii) the loss after a deviation is different for two periods of identical demand, if they differ in location on the business cycle. The most-collusive pricing predictions depend on the capacity costs and fall into two categories. Our main pricing result is with regards to how prices compare on either side of the demand peak. If the marginal cost of capacity is high enough, pricing in two periods with the same demand will be at least as small (much of the time smaller) in the boom than in the recession. While, if the marginal cost of capacity is low enough, pricing in two periods with the same demand will be at least as large (much of the time larger) in the boom than in the recession; the interval of discounts where this is true grows towards the interval as the cost of capacity decreases towards zero. Finally, there is the possibility of a third region of costs in the middle where no blanket pricing patterns are true when comparing booms and recessions.

The Bertrand price competition model in Haltiwanger and Harrington (1991) predicts that collusive prices are weakly lower in similar demand periods in recessions than in booms. The fact that, in our model, all pricing implications are not the same for all capacity costs, highlights the importance of this feature as a determinant of collusive pricing in any industry. The findings in this paper endorse the idea that cyclical variation in pricing is dependent on the expectation of future demand as suggested by Haltiwanger and Harrington, with the caveat that how these prices vary over the cycle depends heavily on the long-run capacity cost in an industry.

The assumptions underlying our model can be extended in a number of directions. For one, alternative rationing rule can be considered. Second, we have assumed that capacity investment takes place at the beginning of the boom. This modeling specification is not entirely innocuous. Gains and losses from capacity deflection would change if the firms were to choose capacities at a different point in the cycle. Hence, choosing capacity in the beginning of the boom is likely to lead to lower aggregate capacities than if firms make this decision later in the boom. A related question we would like to explore further is: when will colluding firms most want to invest? Finally, one could imagine that capacities might be altered after the pricing game begins. For example, firms may alter their capacities in response to observing cheating in the capacity or pricing stages of the game.

23 This calculation is the profit at the non-cooperative symmetric candidate capacities. This provides an approximate average profit from asymmetric equilibria. These capacities are also used for the calculation of most-collusive prices at the non-cooperative capacities. See Lepore (2008) for the characterization of the symmetric candidate capacities.
Appendix A

A.1. Appendix specific notation

Demand is rationed using the surplus maximizing or efficient rationing rule, which is specified in (9).

\[ D_i(p_i, \theta_i) = \begin{cases} z_{i1}(p_i) & \text{if } p_{i1} < p_{i2} \\ \min\{X_iD(p_i, \theta_i) / 2\} & \text{if } p_{i1} = p_{i2} \\ z_{i2}(p_i) & \text{if } p_{i1} > p_{i2}. \end{cases} \]  

(9)

The primary and residual demand functions \( z_{i1}(p_i) \) and \( z_{i2}(p_i) \) are defined as follows,

\[ z_{i1}(p_i) = \min\{x_i, D(p_i, \theta_i)\}, \]
\[ z_{i2}(p_i) = \min\{x_i, \max\{D(p_i, \theta_i) - z_{i1}(p_i), 0\}\}. \]

A.2. Proofs

Proof of Proposition 1. Part 1). In the first part of the proof we show there exists a unique monopoly solution. This part of the proof is broken down into four steps. First we characterize each maximization problem of the pricing subgames. Based on the Assumptions 1–4, for all \( X \in [0, D_{\text{max}}] \), we show there is a unique solution to the problem constrained maximization problem

\[ P^m_t(X) = \arg \max_{P \in \delta(D, \theta)} \{P \cdot D(p, \theta)|D(P, \theta) \leq X\} \]  

for any subgame. If \( \theta \) is such that \( D(P, \theta) < X \), then the solution to the maximization (10) problem is the solution to the unconstrained problem \( P^m_{\text{un}} \) and the revenue is uniquely equal to \( P^m_{\text{un}} \cdot D(P^m_{\text{un}}) \). If the constraint binds, then \( D(P^m_t(X), \theta) = X \) and the price is uniquely determined by this equality by inverting the demand function \( P^m_t(X) = P(X, \theta_t) \). Thus, the monopoly revenue of any period \( t \) is given by

\[ R^m_t(X, \theta_t) = \begin{cases} \frac{P^m_t \cdot D(P^m_t, \theta_t)}{P(X, \theta_t) X} & \text{if } D(P^m_t, \theta_t) > X \\ P(X, \theta_t) X & \text{if } D(P^m_t, \theta_t) \leq X. \end{cases} \]  

(11)

It is straightforward from (11) to see that \( R^m_t(X, \theta_t) \) is bounded and concave in \( X \) for all \( X \geq 0 \), and strictly concave in \( X \) for \( X \in [0, D_{\text{max}}] \).

Denote \( R_t^m(X, \theta_t) = \sum_{i=1}^{\tau} \delta_i R^m_i(X, \theta_t) \). Because \( \theta_t \in (0, 1) \), for all \( t \geq 1 \) and each \( R^m_t(X, \theta_t) \) is bounded, the weighted sum \( R_t^m(X, \theta_t) \) is also bounded. The weighted sum of concave functions is a concave function, therefore \( R_t^m(X, \theta_t) \) is concave on \( X \geq 0 \). We need to show that \( R_t^m(X, \theta_t) \) is strictly concave on \( X \geq 0 \), \( R_t^m(X, \theta_t) \) is strictly concave on \( X \in [0, X_t] \). If

\[ R^m_t(AX + (1 - \lambda)X_\theta, \theta_t) > \lambda R^m_t(X, \theta_t) + (1 - \lambda) R^m_t(X', \theta_t) \]
\[ + (1 - \lambda) R^m_{\tau - 1}(X', \theta_t) \]

where \( \lambda \in [0, 1] \) and \( X, X' \in [0, X_t] \), we can re-write this expression as

\[ R^m_t(AX + (1 - \lambda)X, \theta_t) + R^m_{\tau - 1}(AX + (1 - \lambda)X, \theta_t) \]
\[ > \lambda R^m_t(X, \theta_t) + \lambda R^m_{\tau - 1}(X, \theta_t) + (1 - \lambda) R^m_{\tau - 1}(X, \theta_t) \]
\[ + (1 - \lambda) R^m_{\tau - 1}(X, \theta_t). \]

Based on the concavity of \( R_{\tau - 1}(X, \theta_t) \) on \([0, X_t]\), the expression reduces to

\[ R^m_t(AX + (1 - \lambda)X, \theta_t) > \lambda R^m_t(X, \theta_t) + (1 - \lambda) R^m_t(X', \theta_t) \]
\[ \text{for all } X, X' \in [0, X_t] \]

which is true by the strict concavity of \( P(X, \theta_t) \).

Finally, we show that the profit function of the cycle is strictly concave and bounded, i.e., the maximum exists and is uniquely labeled as \( (X^m, (P^m_t)^{\tau - 1}) \).

The single cycle monopoly profit \( P^m_t(X, \theta_t) = R^m_t(X, \theta_t) - \sum_{i=1}^{\tau} \delta_i c_i X_i \) is concave and bounded on all \( X \geq 0 \). This is enough to guarantee the existence of a maximum, but for uniqueness we first show that any \( X^m \) is not a maximum of \( P^m_t(X, \theta_t) \). Suppose that \( X^m \) is a maximum of \( P^m_t(X, \theta_t) \), then \( \pi^m_t(X^m, \theta_t) \geq \pi^m_t(X^m, \theta_t) \). Note that \( R^m_t(X^m, \theta_t) \) is concave, then that cost function must be such that \( \sum_{i=1}^{\tau} \delta_i c_i X_i \leq \sum_{i=1}^{\tau} \delta_i c_i X_i \). This is a contradiction because, \( \sum_{i=1}^{\tau} \delta_i c_i X_i \) is an increasing function. It follows that the monopoly capacity choice problem can be reduced to

\[ X^m = \arg \max \{\pi^m_t(X, \theta_t)|X \in [0, X_t]\}. \]

On \( X \in [0, X_t] \), \( \pi^m_t(X, \theta_t) \) is strictly concave and the constraint defines a convex and compact set (closed and bounded subset of \( \mathbb{R} \)). Therefore, the maximum \( X^m \) is unique, and from the initial argument of the proof, there is a unique monopoly price determined by \( X^m \).

\[ P_t^m = P^m_t(X) \]  

for all \( t \).

Part 2). In the second part, we show that joint profit maximizing pricing corresponds to constrained-monopoly pricing with capacity \( X = x_1 + x_2 \). First, based on the strict concavity of the function \( P_t \cdot D(P_t, \theta_t) \), we know that among symmetric prices \( p_{1t} = p_{2t} = P_t^m(x_1 + x_2) \) are the unique maximizers.

We are left to show that there does not exist an asymmetric price vector that yields more revenue. Suppose to the contrary that there is an asymmetric price vector that yields more joint revenue for some time period \( t \). Take any asymmetric price vector \( p_0 \), where without loss of generality \( p_1 < p_2 \). There are three cases of demands depending on how the capacities relate to the demand parameter.

Case 1. \( z_{i1}(p_1) = D(p_1, \theta_t) \), then it must be that \( z_{i1}(p_1) = 0 \). The joint revenue for that period is \( p_1 D(p_1, \theta_t) \). If \( p_1 D(p_1, \theta_t) \leq P_t^m D(P_t^m, \theta_t) \) then \( P_t^m D(P_t^m, \theta_t) \) and \( P_t^m D(P_t^m, \theta_t) \) hence for this case, no asymmetric price vector can increase profit.

Case 2. \( z_{i1}(p_1) = x_1 \) and \( z_{i2}(p_1) = x_2 \). The joint revenue for this period is \( p_1^t + p_2^t x_2 \). For this to be true \( p_1^t \leq P_t^m(x_1 + x_2) \), otherwise \( z_{i2}(p_1) = D(p_1, \theta_t) - x_2 \). The following line of inequalities follows from this fact, \( p_1^t \leq x_1 + p_2^t x_2 \leq P_t^m(x_1 + x_2) \) and \( P_t^m(x_1 + x_2) \) hence for this case, no asymmetric price vector can increase profit.

Case 3. \( z_{i1}(p_1) = x_1 \) and \( z_{i2}(p_1) = D(p_2, \theta_t) - x_1 \). The joint revenue for that period is \( p_1^t \leq x_1 + p_2^t x_2 \leq P_t^m(x_1 + x_2) \) and \( P_t^m(x_1 + x_2) \) hence for this case, no asymmetric price vector can increase profit.

Thus, \( p_{1t} = p_{2t} = P_t^m(x_1 + x_2) \) are the joint profit maximizing prices.

Part 3). The final part of the proposition follows immediately from parts 1 and 2 and the fact that the marginal cost of capacity is a constant, \( c \), for both firms. Any \( x \) such that \( x_1 + x_2 = X^m \) results in joint profit \( \pi^m_t(X, \theta_t) \). □

Proof of Proposition 2. Notice that \( R_t^m(p_t, \theta_t) = p_{t1} \min(x_1 + x_2, D(p_{t1}, \theta_t)) \) on the set of symmetric prices, and that \( R_t^m(p_t, \theta_t) \) is a continuous function on these prices. Hence, each function \( R_t^m(p_t, \theta_t) \) is both bounded and continuous on \( P_t \). Since the function \( R_t^m(p_t, \theta_t) \) is the discounted sum of the functions \( R_t^m(p_t, \theta_t) \), \( \forall \theta \in (0, 1) \) it is bounded and continuous.
on \( P(x) \). The set \( P \) is a compact metric space, hence by Tychonoff’s product theorem the set \( P \) is compact. The constraint set is composed of linear weak inequalities of functions continuous in \( p \in P(x) \), hence the relevant constraint set \( \Delta(x, \delta) \) is closed. By construction \( \Delta(x, \delta) \) is a subset of the space \( P \), and a closed subset of a compact space is compact. Since \( (p_i^b(x))_{i=1}^{\infty} \in \Delta(x, \delta) \), the set is non-empty. A continuous function on a non-empty compact set always attains a maximum, therefore \( p_i^b(x) \) exists.

**Proof of Proposition 3.** First we show that given the fixed capacities \( x \), there exists \( \delta_i^b(x) \in (0,1) \) such that \( p_i^b(x) = p_i^b(x) \) if and only if \( \delta \in [\delta_i^b(x), 1] \). We fix \( x \), and analyze how the discount affects the gains \( g_{i,t}(p_i^b(x)) \) and the losses \( L_{i,t}(p_i^b(x)) \), for each firm \( i \) and period \( t \). It is clear that \( L_{i,t}(p_i^b(x)) = g_{i,t}(p_i^b(x)) \), and, \( L_{i,t}(p_i^b(x)) \rightarrow \infty \geq g_{i,t}(p_i^b(x)) \). Note the derivates of the loss and gain function, in terms of \( \delta \), have the following characterizations; \( \frac{dc_{i,t}(p_i^b(x), \delta)}{d\delta} < 0 \) and \( \frac{dc_{i,t}(p_i^b(x), \delta)}{d\delta} = 0 \). Based on the continuity of \( c_{i,t}(p_i^b(x), \delta) \) in \( \delta \), there exists a \( \delta_i^b(x) \) such that \( L_{i,t}(p_i^b(x), \delta) \geq g_{i,t}(p_i^b(x)) \) if and only if \( \delta \in [\delta_i^b(x), 1] \) for each \( t \). This holds for all periods over a single cycle \( t \in \{1,2,...,T \} \) and only if \( \delta \in [\delta_i^b(x), 1] \), where \( \delta_i^b(x) = \min_{x \in X} \delta_i^b(x) \). Therefore, for any \( \delta \), \( p_i^b(x) = p_i^b(x) \) for all \( x \). Define the set \( D(c) = \min \{ \delta \in [0,1] | \delta_i^b(x) \leq \delta, p_i^b(x) \Delta(x, \delta) \} \).

Note that by construction \( \bar{D}(x, \delta) \subset \Delta(x, \delta) \subset [0,1] \), and the set of equilibrium capacities is closed continuous in \( \delta \) at prices \( p \). A closed subset of a compact set is compact, therefore \( D(c) \) is compact and the min \( \min \{ D(c) \} \) exists. Define the discount factor \( \tilde{\delta}(x) = \min D(c) \). This discount factor must be weakly less than \( \delta \), thus \( \tilde{\delta}(x) \leq \delta \) and \( \tilde{\delta}(x) \) satisfies the statement of the proposition.

**Proof of Proposition 4.** For all \( x \in X \), define the term

\[ \delta_i^b(x) = \sup \{ \delta \in [0,1] | \delta_i^b(x) \leq \delta, \Delta(x, \delta) \} \]

Label \( \delta = \min_{x \in X} \delta_i^b(x) \), and

\[ D(c) = \sup \{ \delta \in [0,1] | \delta_i^b(x) \leq \delta, \Delta(x, \delta) \} \]

is non-empty because \( [0,1] \) must be in the set. Define the discount \( \tilde{\delta}(x) = \sup D(c) \) and notice that it meets the criteria of the statement of the proposition.

**Proof of Proposition 5.** Part 1. We know that most-collusive pricing must be pure and symmetric; we must show that they are unique. Note that based on Proposition 1, we know that the revenue function for each \( t \) has the unique pure price maximum \( p_i^b(x) \). Also notice that each function \( R(P_{tx}) = p_i^b(x) \min_{x_i} D(p_i^b(x)) \) is strictly concave on \( P_{tx} \) which implies that for any \( x \in X \), \( R(p_i^b(x)) > \max_{p_i^b(x)} \) if and only if \( p_i^b(x) > \max_{p_i^b(x)} \) or \( p_i^b(x) < \min_{p_i^b(x)} \).

Now suppose to the contrary that there are multiple maximizers. The condition that \( \delta_i^b(x, \delta) \neq 0 \) implies that the maximizer must be a pure symmetric strategy, and not non-collusive pricing. Denote by \( \tilde{P} \), the set of pure strategy maximizers to (6) and by \( \hat{P} \), the set of all prices for period \( t \) that are part of a maximizing price vector \( \forall t \in \{1,2,...,T \} \). Denote by \( \hat{p}_{i,t} \) the pure strategy price vector where each \( \hat{p}_i \) is defined by

\[ \hat{p}_i = \arg \max \{ R_i(p_i^b(x), p_i^b(x)) \} \]

Since \( \hat{p}_{i,t} = \hat{p}_i \leq p_i^b(x) \), the higher the symmetric prices are in the subgame at time \( t \) the harsher the loss from defection is in every other pricing period. The price vector \( \hat{p}_{i,t} \) yields at least as much future loss as the price vectors in \( \hat{P} \), \( \forall t \in \{1,2,...,T \} \). Hence, the incentive constraints for each \( t \) must hold \( \forall \hat{p}_t \), which implies that \( \hat{p}_{i,t} \in \Delta(x, \delta) \). By the construction of \( \hat{p}_{i,t} \), the total revenue in each period is maximal among \( \hat{P} \). This is because each period \( t \) revenue function is single peaked and strictly decreasing away from the peak in the set of symmetric prices. Thus \( R(\hat{p}_{i,t}) > R(p_i^b(x), \forall \hat{p}_t \in \hat{P}_t \). Therefore, the total revenue of \( \hat{p}_{i,t} \) exceeds that of any other \( p_i \), \( \forall p_i \in \hat{P} \), i.e., \( R(\hat{p}_{i,t}) > R(p_i^b(x), \forall \hat{p}_t \in \hat{P}_t \). This contradicts the optimality of any non-single set of strategies \( \hat{P} \).

Part 2. This follows immediately from the definition of \( D_i^b(x, \delta) \). Part 3. Notice that given capacities such that \( x_1 + x_2 \leq D(p_i^b(x)) \) at the symmetric pure strategy price \( p_i^b(x) = \min_{x_i} D(p_i^b(x)) \) for \( i \in \{1,2 \} \) there is no gain from a defection. Hence, for any punishment level \( p_i^b(x) = \min_{x_i} D(p_i^b(x)) \) for \( i \in \{1,2 \} \), is incentive compatible. It follows from Proposition 1 that no other price can lead to joint revenue that is higher.

Before the proof of the Proposition 6 we provide the proof of Lemma 1.

**Proof of Lemma 1.** Let us show that at any symmetric prices the binding incentive constraint is always that of the firm with larger capacity. Without loss of generality we impose that \( x_1 \geq x_2 \).

With symmetric pricing the equilibrium expected revenue for the larger firm is weakly larger in each period. The maximal gain from a defection is then trivially weakly larger for the larger firm as well.

The future losses are the discounted sum of the future revenue minus the punishment. The primary content of the proof is concerned with showing that, for punishments between non-collusive reversion and max−min reversion, future losses to firm 2 are always greater than the future losses to firm 1. We first show that for any period \( t \) and any pure symmetric pricing \( p_t \), the losses from non-collusive reversion punishment is greater for firm 2 in that period.

Notice at any price \( p_{1,t} = p_{2,t} = p_t \), the collusive revenue of each firm is

\[ R_1(p_t, x_1) = p_t \cdot \min \{ x_1, D(p_t, \theta) / 2, D(p_t, \theta) - x_2 \} \]

\[ R_2(p_t, x_1) = p_t \cdot \min \{ x_2, D(p_t, \theta) / 2 \} \]

Label \( L(p_t, x_1) = R_1(p_t, x_1) - R_2(p_t, x_1) \) the non-discounted loss from the price \( p_t \) at period \( t \).

We break this part of the proof into three cases.

**Case 1.** \( p_{1,t} < p_{2,t}(x_t, \theta_t) \). Then \( R(p_t) < R(p_t, x_t, \theta_t) \), which implies that a switch to non-collusive pricing would increase both firms’ revenue in period \( t \) and increase the expected loss in any periods \( s < t \). As a result, all such periods \( s \) can sustain weaker revenue under non-collusive pricing in period \( t \). Hence, \( p_t \) cannot be on the most-collusive equilibrium pricing path.

**Case 2.** \( p_{1,t} < p_{2,t}(x_t, \theta_t) \). Then \( R(p_t) < R(p_t, x_t, \theta_t) \) which is only possible if \( p_{1,t} < p_{2,t}(x_t, \theta_t) \). Then \( R_1(p_t) \geq R_2(p_t) \) and \( R_1(p_t) \geq R_2(p_t, x_t, \theta_t) \) which immediately implies that \( L_2(p_t, x_t, \theta_t) \geq L_1(p_t, x_t, \theta_t) \).

**Case 3.** \( p_t \geq p_{2,t}(x_t, \theta_t) \). There are three subcases we must consider. (i)−(iii).

(i) Suppose that \( p_t \) is such that \( R_1(p_t) = R_2(p_t) \). This implies that \( R_2(p_t) \neq p_t \). The expected revenue of each firm must be no less than the expected revenue from non-collusive reversion. This is because the gain from a defection (zero for both firms) is unaffected, while losses in that period are reduced. Thus, the collusive profit in
period \( t \) and \( s \leq t \) are reduced. Since the expected revenue of each firm must be no less than the non-cooperative revenue expected revenue, \( P_t \geq P(x_1 + x_2, \theta_1) \). But since \( x_1 + x_2 \leq D(\theta_1, \theta_2) \), \( P_t \leq P(x_1 + x_2, \theta_2) \). Hence \( P_t = P(x_1 + x_2, \theta_1) \) and \( L_1, p(x) = 0 \) for both firms.

(ii) Suppose that \( P_t \) is such that \( R_1(p_t, x) = R_2(p_t, x) = P_t(D(p_t)/2 \).

We know that the non-cooperative expected revenue for the larger firm is always weekly greater, thus \( L_1, p(x) \leq L_2, p(x) \).

(iii) Suppose that \( p_t \) is such that \( R_1(p_t, x) = R_2(p_t, x) = P_t(D(p_t)/2 \).

Each firm must receive at least their non-cooperative revenue for period \( t \). The non-cooperative revenue of firm 1 is the maximal residual demand profit, thus the only admissible price is such that \( R_1(p_t, x) = \text{max} \{R_i(p_t, x) \} \).

These three cases cover all possibilities, therefore \( L_1, p(x) \leq \text{max} \{R_1, p(x) \} \) for all \( t \). This clearly implies that for all \( t \), \( L_1, p(x) = \text{max} \{R_1, p(x) \} \).

Now suppose that the firms instead utilized a punishment path stronger. Notice for any punishment path between non-cooperative reversion and min–max reversion, only firm 2 will receive harsher punishments in any period. Hence, \( L_2, p(x) \leq \text{max} \{R_1, p(x) \} \).

When \( p_t = p_t^* \) and \( p_t = p_t^* \), the gain inequality must be strict:

\[
L_1, p(x) < \text{max} \{R_i(p_t, x) \}.
\]

Proof of Proposition 6. Now we give the procedure of the statements of the proposition. We begin by proving that the most-collusive capacities are symmetric. Suppose to the contrary that \( \delta = \delta(x_1, x_2) \) is not \( \geq 0 \) and \( x_1 = x_2 \). If firm 2 increases its capacity to \( x_2 \), then there is weakly greater losses from punishment inflicted to firm 1 in every period. Therefore, \( x' \) it is not symmetric.

Define \( x' \) such that \( x_1 = x_2 = x_1^* \). If firm 1 increase its capacity to \( x_1 \), firm 2 then has weakly less gain in every period for firm 1, therefore \( x = \delta(x_1, x_2) \). If \( \Pi_1(p_t^*, x_1, x_2) > \Pi_1(p_t^*, x, x_1) \), then \( x' \) is increased to \( x_1 = x_2 = x_1^* \).

These two cases together imply the incentive constraint for the most-collusive capacity choice is violated for all \( X = X^* \).
\( \varepsilon > 0 \), such that (13) is satisfied, for all \( c \in [0, \varepsilon(\delta))] \). This implies that, for all \( x_i \in X_i \),

\[
R_i \left( p^*, (X_i, 0), \delta \right) = \sum_i \delta^{-1} c_i X_i < R_i \left( (X_i, 0), \delta \right) = \sum_i \delta^{-1} c_i X_i,
\]

\[
R_i \left( p^*, (X_i, 0), \delta \right) \leq \sum_i \delta^{-1} (X_i - x_i).
\]

(14)

The right-hand side of (14) is continuous, increasing in \( c \), and goes to zero as \( c \to 0 \). For all \( c \in (0, \varepsilon(\delta(\delta))] \) and \( \delta \geq \delta(X, X_0) \), the left-hand side of (14) is strictly positive. Hence, there exists \( \delta > 0 \), such that (13) is true, for all \( c \in (0, \varepsilon) \), since a contradiction of (14) implies (13).

Note that \( R_i(p^*, (X_i, 0), \delta) \) and \( R_i(X_i, X_0, \delta) \) are continuous in \( \delta \), and for \( \delta > \delta(\delta) \), \( R_i(p^*, (X_i, 0), \delta) = \max_{x_i \in [0, \max X_i]} R_i(X_i, X_0, \delta) \). Define \( \delta \) such that for all \( \delta \geq \delta(\delta) \),\( R_i(p^*, (X_i, 0), \delta) \geq R_i(X_i, X_0, \delta) \), for all \( x_i \). For all for all \( c \in (0, 2) \) there exists \( \delta(\delta) > \delta \) such that for all \( \delta \geq \delta(\delta) \), \( \delta(\delta) \geq \sup \{ c \in (0, \varepsilon(\delta))] \} \), and it \( \delta \) satisfies the statement of the theorem. \( \square \)

References


