Experiment 7
Acceleration on an Inclined Plane

In this experiment you study the motion of an object undergoing constant acceleration and measure the acceleration due to gravity, \( g \). The object to be studied, shown in Figure 1, is a cart rolling on low friction wheels and bearings. Note the “flag” (small rectangle) on top of the cart.

The cart is released from rest and allowed to roll down an inclined plane. Its final velocity is measured with a photogate consisting of a photocell and timer. The timer is activated when the “flag” attached to the cart crosses through the photogate.

Preliminaries.

The equation relating the final velocity \( v \), initial velocity \( v_o \), and distance traveled \((x-x_0)\) of an object undergoing constant acceleration \( a \) is

\[
v^2 = v_o^2 + 2a(x-x_0).
\]  

(eq. 1)

The \( x \) axis is aligned along the incline. In this experiment, \( v \) is the cart's average velocity as it passes the photocell located at position \( x \); \( v_o = 0 \) is the starting velocity of the cart at location \( x_0 \). You are to experimentally determine if this equation adequately represents the motion of the cart down the inclined plane.

The measurement of cart velocity is performed at the photogate. As a cart rolls past a photocell the flag it carries interrupts an infrared beam, and this in turn starts an electronic timer displayed on the computer. The timer stops once the beam is again uninterrupted. The timer reading, \( \Delta t \), is the time the cart needs to roll a distance equal to the width of its flag, \( \Delta w \). The cart's average velocity while blocking the photocell is thus

\[
v = \frac{\Delta w}{\Delta t}.
\]

Be careful! A common mistake is to set \( v=(x-x_0)/\Delta t \). This is wrong! This quantity has no physical meaning since \( \Delta t \) is not the time it takes the cart to travel distance \((x-x_0)\).

The smaller the width of the flag the closer the computed value of \( v = \Delta w/\Delta t \) approaches the instantaneous final velocity of the cart.

The track is inclined at an angle \( \theta \) to the horizontal. In this case, \( a_x = g \sin \theta \) if no force except gravity accelerates the cart. The distance the cart travels is \( s = |x - x_0| \). Substituting into the above eq. 1 gives

\[
v^2 = (2g \sin \theta) s.
\]

The term \( 2g \sin \theta \) is constant for any fixed value of \( \theta \).

If this theory correctly applies to the situation you are observing on the track then a plot of \( v^2 \) versus \( s \) will yield a straight line which passes through the origin with a slope equal to \( 2g \sin \theta \).

Procedure.

- Prepare the timer. Turn on the computer and double-click on the icon on the desktop called photogate. A display appears and the timer is ready to be activated.

- Prepare the track. Carefully set the track horizontal by using a long carpenter's level. Then place a spacer under one of the track’s legs.
to raise one end of the track. Use trigonometry to determine the track’s slope angle, θ.

- Determine the final position of the cart. The position $x$ of the cart is its position as it passes the photocell. Unfortunately, it is rather difficult to determine this position precisely. One way to make this measurement is to move the cart by hand very slowly past the photocell and record the position of the cart when the timer just starts ($x_i$) and also when the timer just stops ($x_f$). You can use any point on the cart (such as the rear edge) as a marker to read the position on the scale mounted along the side of the track. The difference between $x_f$ and $x_i$ is the effective width of the flag, $Δw$, as seen by the photocell: $Δw = |x_f - x_i|$. The midpoint between $x_f$ and $x_i$ is the position $x$ at the photocell.

- Let the cart roll a distance $s$ by starting from some position $x_0$. Do several trial runs from the same $x_0$ to make certain that the photogate timer readings $Δt$ give consistent results. Take the average of the trials as the best value. Take the spread in the timer readings as the uncertainty in $Δt$.

- Determine the final cart velocity, $v$.

- Repeat the measurements for at least five very different values of $x_0$ to give a useful graph.

- Graph your data on a $v^2$ versus $s$ plot ($v^2$ on the vertical axis and $s$ on the horizontal axis). Did you obtain a straight line graph? If so, determine the total $x$-acceleration, $a_x$, from the slope of the line, using slope = $2a_x$.

- Determine the acceleration due to gravity. If no friction is present, $a_x = g \sin \theta$. Find the acceleration due to gravity, $g$, and compare your experimental value with the commonly accepted value of $9.80 \text{ m/s}^2 = 980 \text{ cm/s}^2$. The discrepancy is the difference between your measured value and the accepted value. Find the percent difference from

$$\frac{g_{\text{meas}} - g_{\text{accepted}}}{g_{\text{accepted}}} \times 100.$$  

**Questions** (Answer clearly and completely).

1. Does your data indicate that the cart is moving with constant acceleration. How do you know by examination of your graph?

2. If you had graphed the final velocity $v$ versus $s$ (rather than $v^2$ versus $s$), would the data lie on a straight line?

3. What value do you determine for the acceleration due to gravity? (Make sure that you include units!) What is your percent difference from the accepted?

4. The analysis you performed ignored friction. By ignoring friction, do you expect that the value you determined for the acceleration due to gravity is too high? too low? unaffected?