Math 241 Sample Problems for Exam 2

**Question 1** Use Lagrange multipliers to find the maximum and minimum values of \( f(x, y, z) = x - 2y + 5z \) on the sphere \( x^2 + y^2 + z^2 = 30 \).

**Question 2** Evaluate the following double integral:

\[
\int_0^2 \int_{y/2}^1 ye^x \, dx \, dy
\]

**Question 3** Find the volume of the solid in space which lies below the surface \( z = 3 + \cos y \) and above the region in the \( xy \)-plane bounded by the curves \( x = \pi \), \( y = 0 \), and \( y = 2x \) by evaluating an appropriate double integral.

**Question 4** Let \( R \) be the solid region bounded by the planes \( x = 0 \), \( y = 0 \), \( z = 2 \), and the paraboloid \( z = x^2 + y^2 \), in the first octant. Compute \( \iiint_R x \, dV \).

**Question 5** Find the volume determined by \( z \leq 6 - x^2 - y^2 \) and \( z \geq \sqrt{x^2 + y^2} \).

**Question 6** Convert the integral

\[
\int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{2-y^2}}^{\sqrt{2-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{1-x^2-y^2}} z^2 \, dz \, dx \, dy
\]

to spherical coordinates. Don't evaluate it.

**Question 7** Evaluate \( \iint_R \exp\left(\frac{y-x}{y+x}\right) \, dA \) where \( R \) is the triangle with vertices \((0, 0)\), \((1, 0)\), \((0, 1)\), by using the change of variables \( x = \frac{1}{2}(v-u) \), \( y = \frac{1}{2}(u+v) \).

**Question 8**

a) Evaluate the line integral \( \int_c x^2y \, dx + xy^3 \, dy \), where \( c \) consists of the line segments from \((0, 0)\) to \((3, 3)\) and from \((3, 3)\) to \((0, 3)\).

b) Evaluate the line integral \( \int_c (ey + ye^x) \, dx + (e^x + xe^y) \, dy \), where \( c \) is the part of the graph \( y = \ln 6x \) joining \((1/6, 0)\) to \((1/2, \ln 3)\).

**Question 9** Find the area of the surface that is part of the sphere \( x^2 + y^2 + z^2 = 4z \) that lies inside the paraboloid \( z = x^2 + y^2 \).