In answering this problem, show all work. It is your obligation to show clearly how you arrived at your solution—that is, the complete train of logic that leads to your answer.

Points:

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1. The Bode diagram below shows the asymptotes of an open-loop transfer function on the magnitude plot.

   a. What is the transfer function?

   \[ G_{ol} = \frac{10}{(0.1s+1)(0.01s+1)} \]

   \[ 20\log k = 20, \ k = 10^1 = 10 \]

   \[ \omega_m \] is infinite

   \[ \omega_m (\text{gain crossover freq}) \]

   \[ \omega_m \] is infinite
b. Draw the phase plot for this transfer function on the phase diagram above. Label the vertical axis of the phase plot with appropriate values.

c. Mark the gain crossover frequency clearly.

d. Mark the phase crossover frequency clearly.

e. What is the system's gain margin? Show this on the Bode plot.
\[ G_m = \infty \]

f. What is the system's phase margin? Show this on the Bode plot.
\[ \Phi_m = 45^\circ \]

g. What is the system's steady-state error for a unit input? This applies to what kind of input?
\[ e_u = \frac{1}{1 + kp} = \frac{1}{1} \text{ unit step input} \]

h. Now we want to add an integrator to this system to get rid of the steady-state error. Draw the asymptotes of the integrator on the plot above as dashed lines. Draw the modified Bode plots on the grid below (magnitude and phase).
i. Show the new gain and phase margins on the new figure under part h. Is the system stable?

\[ G_m = 20 \text{dB}, \ \phi_m = 45^\circ, \ \text{so stable} \]

j. For the new system with the integrator, what is the finite steady state error for a unit input. For what kind of input is this valid?

\[ k_v = 10, \ \epsilon_M = \frac{1}{k_v} = 0.1 \text{ for ramp input} \]

k. Now add a first-order lead with a break frequency of 1 rad/sec. Draw the asymptotes for this lead on the Bode plots above as dashed lines. Plot the newly modified system on the Bode plot below.
Draw the new gain and phase margins on this plot. Is the system stable?

Yes, system is stable. \( G_m = \infty \), \( \phi_m = 45^\circ \)

What are the controller gains of the controller comprised of the integrator and the first-order lead? I.e., what are \( K_p \), \( K_i \), and \( K_D \) of this controller?

\[
G_c = \frac{(s+1)}{s} = \frac{K_D s^2 + K_p s + K_I}{s}
\]

\( K_D = 0, \ K_p = 1, \ K_I = 1 \)