In giving your answer, the answer alone is not enough. Make sure you clearly give your rationale for arriving at the answer. It must be clear to me how you arrive at your answer.

Points:

1. The system above is a two-tank system in a control loop, where the top tank (with the subscript "Tt") flows into the bottom tank (subscript "Tb"). The actuator is a valve, whose gain is \( K_v \). The system is outfitted with a P-only controller with adjustable gain \( K_p \). The purpose of the loop is to control the level in the lower tank (\( \Delta h_b \)). It is a regulator loop, so the control loop works with deviations from a reference or design state. Hence we have \( \Delta h \)’s instead of \( h \)’s.

For this problem, use flows in gpm, heights in inches, time constants in seconds, and valve opening, \( y \), in %-opening (0%-100%). The tanks take in flow and put out height.

a. On the figure above, on each line write the units for that line.

b. What are the units for:

- \( K_p \): \( \%/\text{in} \)
- \( K_v \): \( \text{gpm}/\% \)
- \( K_T \) (both top and bottom tanks): \( \text{in/gpm} \)
- \( T_T \) (both top and bottom tanks): \( \text{sec} \)

c. What is the control loop’s closed-loop transfer function?

\[
\frac{G_{CL}}{D_{cl} + N_{cl}} = \frac{K_p K_v K_T \Delta h}{(T_T s + 1)(T_{Tb} s + 1) + K_p K_v K_T \Delta h}
\]
d. What is the characteristic equation of the system?

\[ T_T + T_B \cdot s^2 + (T_T + T_B) \cdot s + 1 + K_p \cdot K_v \cdot K_T = 0 \]

e. For the purposes of determining the system's steady-state error, what is its "type"?

Explain your answer.

\[ \text{Type} = 0, \quad G_{OL} \text{ has no pure integrator.} \]

For the remaining problems

Now let both time constants be 100 sec and both \( K_T \)s be 0.1 inch/gpm. Let the valve gain be 0.1 gpm/\%.

f. Calculate the limits of \( K_p \) for stability using the Hurwitz criteria. Show units in your calculation.

\[ \text{Since } T_T > 0, \quad 1 - K_p K_v K_T > 0 \]

\[ K_p > -\frac{1}{K_v K_T} = -\frac{1}{0.1 \text{ gpm/\%} 
\quad \times 0.1 \text{ in/gpm}} \]

\[ K_p > -100 \text{ in/\%} \]

\[ K_p > -100 \frac{\text{in}}{\text{\%}} \]

\[ K_p = 200 \text{ \% in} \]

With a unit step input, what would be the steady-state error?

Show units in your calculation.

\[ e_{ss} = \frac{R_o}{1 - K_p - e_{ss}} \quad R_o = 1 \text{ in} \]

\[ K_p - e_{ss} = \lim_{s\to0} \frac{G_{OL}}{s} = K_p K_v K_T \]

\[ = 200 \frac{\text{in}}{\text{\%}} \times 0.1 \text{ gpm/\%} \times 0.1 \text{ in/gpm} \]

\[ = 2 \]

\[ e_{ss} = \frac{1 \text{ in}}{1 + 2} = \frac{1}{3} \text{ in} \]

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h. What are the system's open-loop zeros?

There are none

i. What are the system's open-loop poles?

\[ s = -\frac{1}{100 \text{ sec}^{-1}} \] (two of these)

j. Draw the root locus of this system on the plot below.
### Formulae and tables

\[
G_{cl} = \frac{G}{1+GH} = \frac{D_G D_H}{D_G D_H + N_G N_H} \quad G_1 = \frac{K_{ss}}{s+1} \quad G_2 = \frac{K_{ss} \omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2} = \frac{K_{ss}}{\frac{s^2 + 2\zeta s + 1}{\omega_n}}
\]

<table>
<thead>
<tr>
<th>Input</th>
<th>Type 0</th>
<th>Type 1</th>
<th>Type 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(e_{ss})</td>
<td>(e_{ss}) constant</td>
<td>(e_{ss})</td>
</tr>
<tr>
<td>Step</td>
<td>(\frac{R_0}{1 + K_{p-ess}})</td>
<td>(K_{p-ess} = constant)</td>
<td>(\frac{R_0}{1 + K_{p-ess}})</td>
</tr>
<tr>
<td>Ramp</td>
<td>(\frac{R_0}{K_v})</td>
<td>(K_v = 0)</td>
<td>(\infty)</td>
</tr>
<tr>
<td>Parabola</td>
<td>(\frac{R_0}{K_a})</td>
<td>(K_a = 0)</td>
<td>(\infty)</td>
</tr>
</tbody>
</table>

\(K_{p-ess}, K_v, K_a\)