ME 422 – Quiz 2
Winter 2012

In giving your answer, the answer alone is not enough. Make sure you clearly give your rationale for arriving at the answer. It must be clear to me how you arrive at your answer.

Formulae:
\[ G_1(s) = \frac{K_{ss}}{s^3 + 1} \]

\[ G_2(s) = \frac{K_{ss} \cdot \omega_n^2}{s^2 + 2 \cdot \zeta \cdot \omega_n \cdot s + \omega_n^2} \]

Sufficient Hurwitz conditions for stability:

n = 3: \( a_0 \cdot a_2 - a_1 \cdot a_3 < 0 \)

n = 4: \( a_0 \cdot a_3^2 + a_4 \cdot a_1^2 - a_1 \cdot a_2 \cdot a_4 < 0 \)

n = 5: \( a_2 \cdot a_5 - a_3 \cdot a_4 < 0 \) and \( (a_0 \cdot a_3 - a_1 \cdot a_2)^2 - (a_3 \cdot a_4 - a_3 \cdot a_5)^2 \cdot (a_1 \cdot a_2 - a_0 \cdot a_3) < 0 \)

Steady-state error constants:

\[ K_{p-ess} = \lim_{s \to 0} (G \cdot H), \quad K_v = \lim_{s \to 0} (s \cdot G \cdot H), \quad \text{and} \quad K_a = \lim_{s \to 0} (s^2 \cdot G \cdot H) \]

<table>
<thead>
<tr>
<th>Input</th>
<th>Type 0</th>
<th>Type 1</th>
<th>Type 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \frac{R_0}{1 + K_{p-ess}} )</td>
<td>( R_0 )</td>
<td>( R_0 )</td>
</tr>
<tr>
<td>Step</td>
<td>e_\text{ss} constant</td>
<td>e_\text{ss} constant</td>
<td>e_\text{ss} constant</td>
</tr>
<tr>
<td>Ramp</td>
<td>( \frac{R_0}{K_v} )</td>
<td>K_v = 0</td>
<td>( \frac{R_0}{K_v} )</td>
</tr>
<tr>
<td>Parabola</td>
<td>( \frac{R_0}{K_a} )</td>
<td>K_a = 0</td>
<td>( \frac{R_0}{K_a} )</td>
</tr>
</tbody>
</table>
1. For the system shown at right, answer the following questions.

\[ G_{CL} = \frac{10 N_0 D_4}{D_9 D_4 + N_7 N_{14}} = \frac{10 K_p}{3(s+1) + K_p 5^3} \]

a. What is the closed-loop transfer function for the above system, including the pre-loop gain? Put this into a standard form.

\[ \omega_n = \sqrt{15K_p} \]

b. What is \( \omega_n \)?

c. What is \( \zeta \)?

\[ 25 \omega_n = 1, \quad 5 = \frac{1}{2 \omega_n} = \frac{1}{2\sqrt{15K_p}} = 5 \]

d. What is \( K_s \)? To get this, put in a unit step and FVT.

More room

\[ y_{ss} = \lim_{s \to 0} s G_{CL} \frac{1}{s} = \lim_{s \to 0} G_{CL} = \frac{50K_p}{15K_p} = \frac{10}{3} = 3.33 = K_s \]

Since unit step put in, this is \( K_{ss} \).

e. What is the characteristic equation?

\[ s^2 + s + 15K_p = 0 \]

f. For what values of \( K_p \) is the system stable? Show work.

More room

Since order is 2, necessary condition is sufficient condition is sufficient.

So \( 15K_p > 0 \) \( \iff \) \( K_p > 0 \).

g. What type of input gives a finite steady-state error? Explain.

\[ G_{CL} = \frac{10 \cdot 5 \cdot 3 K_p}{s(s+1)} \] type I system.

Ramp gives finite error, from table on page 1. Go on to next page...
h. Assume this type of input, at a unit level (i.e., \(X(s) = 1/s, 1/s^2, 1/s^3\), or something of this form). What would be \(e_{ss}\) for this unit input?

i. For what values of \(K_p\) will the system oscillate?

\[
\frac{1}{1 + 15K_p} \leq 1 \quad \frac{1}{60K_p} \leq 1
\]

Assuming \(K_p > 0\), \(\frac{1}{60} \leq K_p\)

j. What will the closed-loop poles be when \(K_p = 0\)? Show work.

3. chm. eq: \(s^2 + s = 0\) \(s = 0, 0, -1\)

k. Draw the root locus of this system. (Hint: This system's configuration matches that analyzed in class Wednesday, so \(\zeta, \omega_n \neq f(K_p)\).)

- Diagram showing root locus with annotations:
  - If \(G_{ol} = 150K_p\), \(K_v = 150K_p\)
  - If \(R_0 = 1\) get \(K_v = 150K_p\)
  - \(e_{ss} = \frac{1}{150K_p}\)
  - Wrong answer!
  - Total = 49 pts

\(K_v = \lim_{s \to 0} G_{ol} = 0\) 
\(K_v = \lim_{s \to 0} \frac{K_p}{s(s+1)} \frac{5.3}{s(s+1)} \)
\(K_v = 15K_p\) 
\(e_{ss} = \frac{10}{15K_p}\)
\(e_{ts} = \frac{2}{3K_p}\)