In giving your answer, the answer alone is not enough. Make sure you clearly give your rationale for arriving at the answer. It must be clear to me how you arrive at your answer.

Weights: 1 = 60%, 2 = 40%

1. Above is shown a block diagram of the Motomatic position controller.

   a. Find the open-loop transfer function of the system.
   
   $$G_{OL} = \frac{K_K R_f K_A K_M K_\theta}{R_S (T_M S + 1) S R_B}$$

   b. Find the closed-loop transfer function of the system
   
   $$G_{CL} = \frac{N_G D_H}{D_G D_H + N_G N_H} = -\frac{K_K}{R_S} \frac{R_f K_A K_M R_B}{(T_M S + 1) S R_B - R_f K_A K_M K_\theta}$$

   Continued on back...
c. What is the natural frequency of the closed-loop system in terms of the system parameters given in the block diagram?

\[ G_{cl} = \frac{-R_f K_a K_m R_B K_k}{s^2 + \frac{1}{T_m} s - \frac{R_f K_a K_m K_0}{T_m R_B}} \]

\[ \omega_n = \sqrt{\frac{-R_f K_a K_m K_0}{T_m R_B}} \]

d. What is the damping ratio (\( \zeta \)) of the closed-loop system?

\[ 2 \zeta \omega_n = \frac{1}{T_m} \]

\[ \zeta = \frac{1}{2 \omega_n T_m} = \frac{1}{2 \omega_n} \sqrt{\frac{T_m R_B}{R_f K_a K_m K_0}} \]

\[ \zeta = \frac{1}{2} \sqrt{\frac{R_B}{T_m R_f K_a K_m K_0}} \]

e. For what range of values of \( R_f \) is the system stable? Show work.

\[ \frac{-R_f K_a K_m K_0}{T_m R_B} > 0 \]

All variables > 0 except \( K_a \) & \( K_a < 0 \) so \[ \frac{-K_a K_m K_0}{T_m R_B} > 0 \]

\( R_f > 0 \) as is its inverse.