ME 422 – Quiz 1
Winter 2014

In giving your answer, the answer alone is not enough. Make sure you clearly give your rationale for arriving at the answer. It must be clear to me how you arrive at your answer.

1.

For the system shown above,

a. Calculate the equivalent single block that replaces this network of blocks.

\[
G_{EQ} = \frac{K^2}{(Ts+1)^2} + \frac{K}{Ts+1} = \frac{K^2 + K(Ts+1)}{(Ts+1)^2}
\]

b. What is the steady-state gain of the equivalent system?

\[
K_{st} \rightarrow \infty, \quad S \rightarrow 0, \quad \text{do}
\]

\[
K_{EQ} = K^2 + K
\]

c. What is the equivalent system’s \( \omega_n \)?

\[
\text{In denom: } T^2s^2 + 2Ts + 1 = \frac{1}{\omega_n^2} s^2 + \frac{2b}{\omega_n} s + 1
\]

\[
\omega_n = \frac{1}{T}
\]

d. What is the equivalent system’s \( \zeta \)?

\[
\frac{2b}{\omega_n} = \zeta T, \quad \zeta = T \omega_n = 1, \text{ which it should be because } IR \text{ repeated roots}
\]

e. Is the equivalent system underdamped, critically damped, or overdamped? Explain.

Repeated IR roots ⇒ critically damped
2. In the Laplace solution of an ODE, you arrive at the point:

\[ X(s) = \frac{12}{(s+3)(s+4)} \]

(a) Use partial fraction expansion to solve for \( x(t) \). So all steps

\[ \frac{12}{(s+3)(s+4)} = \frac{A}{s+3} + \frac{B}{s+4} \]

\((s+3): \frac{12}{s+4} = A + \frac{B(s+3)}{s+4}\) let \( s = -3 \)

\[ A = 12 \]

\((s+4): \frac{12}{s+3} = \frac{A(s+4)}{s+3} + B\) let \( s = -4 \)

\[ -12 = B \]

(b) What is the value of \( x(t) \) at \( t = 0 \)? Show work.

\[ x(t) = 12e^{-3t} - 12e^{-4t} \]

(c) What is the value of \( x(t) \) as \( t \to \infty \)?

As \( t \to \infty \), \( e^{-3t} \) & \( e^{-4t} \to 0 \), so \( x(\infty) = 0 \)

(d) Develop an expression for \( x(t) \).

\[ x(t) = -36 e^{-3t} + 48 e^{-4t} \]

e. What is the largest value \( x(t) \) reaches for \( 0 \leq t \leq \infty \)? At what time does \( x(t) \) reach this value?

Set \( x(t) = 0 \) & find \( t \).

\[ 0 = -36 e^{-3t} + 48 e^{-4t} \]

\[ 3 e^{-3t} = 4 e^{-4t} \]

\[ e^{-3t} = \frac{4}{3} e^{-4t} \]

\[ -3t = \ln \left( \frac{4}{3} \right) - 4t \]

\[ t = \ln \left( \frac{4}{3} \right) = 0.2877 \text{ sec} \]

\[ x(0.2877) = 1.266 \text{ max displacement} \]