Answer the problems below showing all work. A correct answer is insufficient for full credit. It must also be clear how you arrived at your answer.

1. A control loop has a P-only controller with a gain of $K$, an actuator with a gain of $1/K$, and a first-order plant with $K/(T_s + 1)$. The sensor is slow so is also modeled as a first-order system with $(K/4)/([T/8]_s + 1)$.

a. Draw the control loop of this system.

![Control Loop Diagram]

b. What is the open-loop transfer function of the control loop?

$$G_{OL} = \frac{K^2/4}{(T_s + 1)(\frac{T}{8} s + 1)}$$

$$= \frac{K^2/4}{\frac{T^2}{8} s^2 + \frac{9T}{8} s + 1}$$

2. What is the open-loop gain?

$$K_{ss-OL} = \frac{K^2}{4}$$

Go on to next page...
d. What is the closed-loop transfer function of the control loop?

\[ G_{CL} = \frac{\frac{T}{8} s + 1}{(\frac{T}{8} s + 1)^2 + \frac{1}{k^2} s + 1} \]

\[ G_{CL} = \frac{K^2 (\frac{T}{8} s + 1)}{K (\frac{T}{8} s + 1) (\frac{T}{8} s + 1) + k^2 K/4} \]

\[ G_{CL} = \frac{1}{\frac{T^2}{8} s^2 + \frac{9}{8} T s + 1 + k^2/4} \]

\[ 2 \delta \omega_n = \frac{9}{4} \] (2)

\[ \delta = \frac{9}{2} \sqrt{2 (4 + k^2)} \]

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\[ K_{ss} = \frac{8 K}{2 (4 + k^2)} \]

\[ K_{ss} = \frac{4 K}{4 + k^2} \]

\[ K > \frac{7}{2 \sqrt{8}} \]

\[ K > \frac{7}{7.17} \]

\[ \left( \frac{81}{8} - 4 \right) < k^2 \] (3)

\[ 81 \left( \frac{1}{8} \right) - 4 < k^2 \]

\[ k^2 > \frac{81 - 32}{8} = \frac{49}{8} \]

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