ME 422
LECTURE 19

Outline

- What is frequency response?
- Log magnitude (dB) and phase angle
- Frequency response of common system components
- Remember, no class on Friday but a quiz covering root locus on Monday
- Chapter 7 solutions on-line
What is frequency response?

- Root locus developed by Evans in 1948
- Frequency response was method used for system analysis and controller design prior to that
  - So all controls development in WWII done with frequency responses
- Frequency response still useful
  - Another perspective
  - Handles some things better than root locus
Use known input signal on black box

One main use of frequency response is to characterize (find the transfer function of) black boxes:

We feed a known signal, $X(s)$, into a black box and watch how it affects this signal—that is, look closely at the output, $Y(s)$

More specifically...

Use sinusoidal input:

If system is linear, output will be sinusoid with same frequency but with a different amplitude and out of phase with the input.
Oscilloscope plot of FR test

On an oscilloscope:

- $x(t)$
- $y(t)$
- Phase shift

Note:
- $X \neq Y$
- Can use time shift to calculate phase shift

Even more specifically...

Frequency Response test:

1. $x_{1\omega_1}$
2. $x_{1\omega_2}$
3. $x_{1\omega_3}$

Typically you keep input amplitude the same and vary (increase) frequency. $Y$ is a function of $\omega$. $\phi$ is also a function of $\omega$. What's going on here? Why did amplitude increase and then decrease?
Output of such a test

- Here the input amplitude was left constant
- $M$ is the ratio $Y/X$
- We are going to plot $20 \log M$, decibels, will be explained soon

<table>
<thead>
<tr>
<th>$\omega$, rad/sec</th>
<th>X</th>
<th>Y</th>
<th>M = Y/X</th>
<th>$20 \log M$, dB</th>
<th>$\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>10</td>
<td>1000</td>
<td>100</td>
<td>40.00</td>
<td>-90</td>
</tr>
<tr>
<td>3</td>
<td>0.05</td>
<td>10</td>
<td>13.5</td>
<td>13.5</td>
<td>22.61</td>
</tr>
<tr>
<td>4</td>
<td>0.1</td>
<td>10</td>
<td>92.3</td>
<td>9.23</td>
<td>19.30</td>
</tr>
<tr>
<td>5</td>
<td>0.5</td>
<td>10</td>
<td>12.2</td>
<td>1.22</td>
<td>1.73</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>10</td>
<td>2.24</td>
<td>0.224</td>
<td>-13.00</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>10</td>
<td>24</td>
<td>2.4</td>
<td>7.60</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>10</td>
<td>9.96</td>
<td>0.958</td>
<td>-0.02</td>
</tr>
</tbody>
</table>

- Plot with logarithmic $\omega$-axis
- Line up $\omega$s on each chart
- These two plots together are known as a Bode Plot of the system
Bode plots of common components

Frequency response tests have been run on all common components (s, 1/s, K/(Ts+1), etc.) and tabulated (see Table 8.1 in text):

<table>
<thead>
<tr>
<th>Component</th>
<th>Bode plot</th>
<th>Characteristics</th>
</tr>
</thead>
</table>
| Gain, K     | 0         | 20 log K
            | 0         | If K > 1, 20 log K > 0
            | 0         | If K < 1, 20 log K < 0
            | 0         | Gain has zero phase. So when it is added to a system, it simply shifts the system’s log mag curve up (K>1) or down (K<1).

Response of a pure gain, K

Response of s and 1/s

- Differentiator, s
  - Has a constant slope upward of 20 dB/dc
  - With s = jω and M = 1, M = 1, so 20 log M = 0.
  - So differentiator crosses – axis of log mag plot at ω = 1.
  - Can recognize system with s in it because no matter how small ω is, log mag plot still has upward slope and θ starts at 90 degrees.

- Integrator, \( \frac{1}{s} \)
  - Has a constant slope downward of -20 dB/dc
  - With s = jω and M = 1, M = 1, so 20 log M = 0.
  - So integrator crosses + axis of log mag plot at ω = 1.
  - Can recognize system with 1/s in it because no matter how small ω is, log mag plot still has downward slope and θ starts at -90 degrees.
Response of $Ts+1$ and $1/(Ts+1)$

These are known as first-order lead and lag

Response of second-order lead and lag

<table>
<thead>
<tr>
<th>Component</th>
<th>Bode plot</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2^{nd}$ order lead, $\frac{1}{s^2 + \omega_n^2 + 2ζ\omega_ns}$</td>
<td><img src="image" alt="Bode plot" /></td>
<td>Starts out with 0 log mag. Asymptote starts a -40 dB/dcd fall at break frequency, which is the natural frequency. Behavior right around break frequency depends on $ζ$. Phase curve starts at 0 and falls to -180 degrees. It is at -90 degrees at break frequency.</td>
</tr>
<tr>
<td>$2^{nd}$ order lag, $\frac{1}{s^2 + \omega_n^2 + 2ζ\omega_ns}$</td>
<td><img src="image" alt="Bode plot" /></td>
<td>Starts out with 0 log mag. Asymptote starts a 40 dB/dcd rise at break frequency, which is the natural frequency. Behavior right around break frequency depends on $ζ$. Phase curve starts at 0 and rises to 180 degrees. It is at 90 degrees at break frequency.</td>
</tr>
</tbody>
</table>
With this table, we can find transfer functions of black boxes

**Example**

Flat log mag curve
At 37 dB

ϕ is 0 for all ω

Top plot is of 20 log(K), not K

\[ 37 \text{ dB} = (20 \text{ dB}) \log(K) \]
\[ \log(K) = \frac{37}{20} \]
\[ K = 10^{37/20} = 70.8 \]
With this table, we can find transfer functions of black boxes

**Example**

- Flat log mag curve at -7 dB
- $\phi$ is 0 for all $\omega$

**Top plot is of 20 log(K), not K**

\[-8 \text{ dB} = (20 \text{ dB}) \log(K)\]

\[\log(K) = -\frac{8}{20} \]

\[K = 10^{-8/20} = 0.3981\]

If 20 log(K) is < 0, K < 1
What about this one?

Log mag curve starts at 0 dB

In 1 decade, descending line drops 20 dB

ϕ = -45° @ 0.05 rad/sec

ϕ = -90° @ high frequencies

Example