Coriolis Acceleration

Besides normal acceleration and tangential acceleration that are found on rotating rigid bodies, there is also Coriolis acceleration that occurs when a mass moves radially on a rotating body. Take the example below of a cockroach on a spinning LP record.

At $t$

The small-letter coordinate system spins with the disk, the big-letter coordinate system is fixed. At a slightly later moment in time, the disk has turned through an angle $\Delta \theta$.

At $t + \Delta t$

$\Delta \theta = \omega \Delta t$
For simplicity, let \( \omega \) & \( V_x \) be constants. Note that \( V_x \) is a radial velocity, so it represents the time rate of change of distance from the origin, \( \vec{r} \).

At \( t + \Delta t \), the cockroach has moved to a slightly larger radius on the record platter. If the platter is spinning at a steady speed, the cockroach will have a higher tangential speed, \( V_y \), at \( t + \Delta t \) because he is further out away from the center of spin.

\[
V_y(t) = \omega r
\]

\[
V_y(t + \Delta t) = \omega (r + \Delta r)
\]

Thus the magnitude of the tangential acceleration due to this effect is

\[
\vec{a}_{\text{tang}} = \frac{V_y(t + \Delta t) - V(t)}{\Delta t} = \frac{\omega (r + \Delta r) - \omega r}{\Delta t}
\]

\[
\vec{a}_{\text{tang}} = \frac{\omega \Delta r}{\Delta t} = \omega \vec{r} = \omega \vec{V}_x
\]

The direction will be in the \( y \) direction, so also tangential. This is not \( \vec{a}_x \), which depends on \( \theta \). This represents an increase in \( V_y \) because the rotation of the disc speeds up. With \( \vec{a}_{\text{tang}} \), \( a \) can be 0 and \( \vec{a}_{\text{tang}} \neq 0 \).
Note that $\vec{a}_{\text{cor-vy}}$ can be constructed using the cross product.

$$\vec{a}_{\text{cor-vy}} = \vec{\omega} \times \vec{V}_x$$

This will lead to $\vec{a}_{\text{cor-vy}}$ of the proper magnitude pointed to the left of the cockroach’s direction of travel. Also, if the cockroach is headed radially inward toward the center, his tangential velocity will be decreasing because the radius is decreasing. With this cross product, $\vec{a}_{\text{cor-vy}}$ points opposite $\vec{V}_y$, which indicates a decreasing $V_y$.

But this is not all. There’s another effect caused by the change of direction of $\vec{V}_x$. If we consider $\vec{V}_x(t)$ & $\vec{V}_x(t+\Delta t)$ from the drawing on page 1,

$$\Delta \vec{V}_x = \vec{V}_x \Delta \theta$$  \hspace{1cm} (Again we consider $V_x$ constant)

$$\Delta \theta = \omega \Delta t$$

$$\vec{a}_{\text{cor-vx}} = \frac{\Delta \vec{V}_x}{\Delta t} = \vec{\omega} \times \vec{V}_x$$
This is exactly what we got for

\[ \vec{a}_{\text{cor}} = \vec{a}_{\text{cor-vy}} + \vec{a}_{\text{cor-vx}} = 2 \vec{\omega} \times \vec{v}_x \]

**Interesting cases**

The codewalker turns &

walks tangentially,

instead of radially.

So his radius is 

no longer changing.

Thus \( \vec{v}_x = 0 \). Obviously the gain in

the magnitude of \( \vec{v}_y \) due to radius,

increase or decrease is absent. But

what about the cor component due
to the change in direction of \( \vec{v}_y \)?

\[ \Delta \vec{v}_y = -\vec{\omega} \times \vec{v}_x \]

\[ \Delta \theta = \frac{\Delta \vec{v}_y}{\vec{v}_x} \]
This is directed inward toward the center and is just centripetal acceleration. The cockroach is merely augmenting his tangential velocity due to the rotation of the disk. So in this case answer, recall also that

\[ a_n = \frac{v^2}{r} \]

But here \( v = w r + v_y \).

We can also see from the drawing that the normal acceleration due to this walking speed is

\[ \Delta v_y = v_y \Delta \theta = v_y w \Delta t \]

So \( a_{n-v_y} = \frac{\Delta v_y}{\Delta t} = v_y w \)

The standard \( a_{n_w} = \overline{\omega} \times (\overline{w} \times \overrightarrow{r}) \)

\[ |a_{n_w}| = w v_w \]

So in this case \( a_n = a_{n_w} + a_{n-v_y} \)

\[ a_n = w v_w + w v_y = w (v_w + v_y) \]

So the added walking speed increases \( a_n \). This makes sense. \( a_n \) is and always due to the change in direction of \( \overrightarrow{v} \).
Coriolis

If the cockroach walks against the direction of rotation, his velocity is diminished.

\[ a_n = w (v_w - v_y) \]

If, in fact \( v_y = v_w \), the cockroach will simply walk in place with the disk spinning under him. He will experience no acceleration at all because he is not moving.

If he heads off in a direction at an angle to the radius \( \phi \),

\[ v_y \]

must be broken into components, one radial and one tangential, and treated separately, as described above.
In this case, at first glance, there is no Coriolis acceleration present. The disk OA spins at a constant speed and a bar at A slides through a pivoting collar at C. But be careful. The standard way of dealing with this—by finding \( \omega_{AB} \) and \( \omega_{AB} \)—is to work from what's known to what's unknown and to use A as a reference point. So \( \vec{v}_A \) and \( \vec{a}_A \) due to \( \vec{w} \) are known, and we write

\[
\vec{a}_D = \vec{a}_A + \vec{a}_{D/A}
\]

For looking at \( \vec{a}_{D/A} \), an observer is standing on A and is completely unaware of his/hers own motion. With A as the reference point, the observer sees the collar sliding up and down AB while AB is rotating. So we have radial motion and rotation, hence Coriolis acceleration.
Here some type of robotic structure has 3 DoF. The disk is spinning.
At the same time the arm is rotating on the disk, and also the collar at B is travelling outward along the rod. Let all velocities \( w_2, \omega_2A, V_B \) be constant. Let \( \vec{x} \) be where the rod projects onto the disk. We are interested in \( \vec{a}_B \). It is best to consider what \( \vec{a}_B \) would be under the action of each rotation separately. So first suppose \( V_B = 0 \) and \( \omega_2 = 0 \). \( \vec{a}_B - \omega_2 \) would be horizontal directed toward the disk spin axis \( z \).

If \( w_2 = 0 \) & \( \omega_2A \neq 0 \& V_B = 0 \), \( \vec{a}_B \) would again be normal and directed down the bar from B toward O.

These two normal accelerations would occur even if \( V_B = 0 \). Coriolis acceleration is trickier because it's the combination of radial velocity & rotation. If \( V_B \neq 0 \) and \( w_2 = 0 \& \omega_2A = 0 \), there would be no acceleration, no \( a_B \).
The motion of \( V_B \) is partially radial & partially vertical, so we need to extract \( V_{Bx} \). Looking from the -y direction:

\[
V_{Bx} = V_B \cos \Theta_{OA} - V_B \omega_{OA} \sin \Theta_{OA}
\]

This would be the velocity to use in the calculation of \( \Delta \alpha_t \). It is the rate at which the radius from the z spin axis is changing. Both of these velocities have upward (z) components, but they play no role in Coriolis or any other kind of acceleration.

So with all 3 velocities non-zero:

\[
\ddot{\alpha}_B = -\omega_z^2 R_{/BO} \cos \Theta_{OA} \hat{z} \\
- \omega_{OA} R_{/BO} (\cos \Theta_{OA} \hat{z} + \sin \Theta_{OA} \hat{k}) \\
+ 2 \omega_z (V_B \cos \Theta_{OA} - V_B \omega_{OA} \sin \Theta_{OA}) \hat{y}
\]

where \( \hat{z}, \hat{y}, \) and \( \hat{k} \) are rotating unit vectors.
In all of this, what would happen if \( \omega \neq 0 \) and/or \( \alpha \neq 0 \)? If you look back through this, you will see that Coriolis acceleration is caused by a rotational and radial velocity. Acceleration has nothing to do with it. \( \omega \) will make \( \mathbf{\Omega} \) change, and \( \alpha \) will make \( \mathbf{\Omega} \) change. But Coriolis acceleration depends on the velocities at the instant it is calculated, and it does not matter whether they are increasing or decreasing.