Modeling of Car Braking with and without ABS
by Frank Owen, PhD, P.E., Mechanical Engineering/Fakultät 03,
Cal Poly/Hochschule München
(revised 13 July 2012)

In this segment of the course we shall look at modeling a common mechatronic system in an automobile, the anti-lock braking system or ABS. You have already seen this in a previous development in this course. In that segment you modeled the car undergoing braking and the ABS system in Matlab. Here we will do something similar except we will use Simulink to do it.

The main problem addressed by an ABS system is the reduced braking force that occurs when a car’s wheels slip on the pavement. The braking force during slip reduces to about half their non-slip value. The purpose of the ABS is to detect slip, then momentarily reduce the braking force to prevent the slip. Drivers of cars without ABS in cold, icy climates were always trained to brake by “pumping” the brakes. This meant never to push hard and steadily on the brake pedal but rather to tap the brakes, to apply a high frequency, drum-beat-sort of force to the brake pedal.

This is essentially what the ABS system does. When slip is detected through sensors, valves are actuated to release the pressure on the brake pads, thus momentarily allow the wheel to roll out of slip. Once out of slip, the valves act to allow the build-up of pressure once again on the brake pads.

In this segment we will look once again at automobile braking and model this situation in an automobile with and without ABS.

The Analysis

In our analysis the car is moving to the right with a velocity $v_C$. The braking forces on the wheels act to the left to slow the car down.

We will simplify the analysis by looking at only one wheel, using what is often called the quarter-car model of the car. Here we concentrate the entire mass of the quarter car at the hub of the wheel.
From a dynamic equilibrium analysis of this wheel, summing moments about point \( O \), the center of the wheel, taking clockwise to be positive:

\[
\sum M_{O} : \quad F_{B-Q} r_{W} - M_{B-Q} = J_{W} \dot{\omega}
\]  

(1)

The maximum possible braking force depends on the slip, specifically on the slip parameter \( s \). \( s \) is a measurement of the actual velocity of the car compared with the kinematically calculated velocity if there were no slip \( (v_{C-\omega} = \omega r_{W}) \).

\[ s \] is the slip. It is defined by

\[
s = \frac{v_{C} - v_{C-\omega}}{v_{C}} = \frac{v_{C} - \omega r_{W}}{v_{C}}
\]  

(2)

\( s \) varies between 0 (no slip) and 1 (full slip). From the slip can be calculated the slip parameter \( f(s) \). A suggested relationship for \( f(s) \) is

\[
f(s) = c_{1}(1 - e^{-c_{2}s}) - c_{3}s
\]  

(3)

The three parameters—\( c_{1}, c_{2}, \) and \( c_{3} \)—can be adjusted to match test data taken on the actual automobile. For the situation presented earlier in the course, we can use the following values:
$f(s)$ is then used to find the braking force:

$$F_{B-Q} = f(s) \mu F_{N-Q} \quad (4)$$

We now expand the quarter-car model and just say that what happens with the entire car is just 4x by what happens on each wheel. If we look at the entire car, we see that we have in the $x$ direction:

where $F_B$ is the total braking force and $F_A$ is wind resistance. We can simplify the analysis to assume that $F_B$ is just $4 * F_{B-Q}$. This is not really correct because the front wheels carry more of the braking load due to the pitching movement of the car during braking. This loads the front wheels and unloads the back wheels, but for this simple analysis, we can proceed with this as an initial approach. So
\[-F_B - F_A = m_C a_C\]
\[a_C = \ddot{x}_C = \frac{1}{m_C} (- F_B - F_A) \quad (5)\]

Usually \(F_A\) varies as the square of the velocity:

\[F_A = c_A A \rho \frac{v_C^2}{2} \quad (6)\]

So

\[\ddot{x}_C = \frac{1}{m_C} \left( - f(s) \mu F_N - c_A A \frac{\rho \dot{x}_C^2}{2} \right) \quad (7)\]

As previously mentioned, when the brakes are applied, the car pitches forward, loading up the front wheels. Thus the front wheels carry more vertical force than the normal weight of the car. This allows for a greater braking force than would be possible if the front wheels saw only their share of the weight of the car. We can account for this extra force by using a multiplier, say 1.5, multiplied by \(F_N\). So

\[F_N = 1.5 * mg \quad (8)\]

**Building the Model**

Use the following values for the simulation:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value/Units</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(v_{C,0})</td>
<td>100 kph</td>
<td>initial speed</td>
</tr>
<tr>
<td>(m)</td>
<td>1500 kg</td>
<td>automobile mass</td>
</tr>
<tr>
<td>(r_w)</td>
<td>0.3 m</td>
<td>wheel radius</td>
</tr>
<tr>
<td>(J_W)</td>
<td>0.8 kg*m^2</td>
<td>mass moment of inertia of wheel</td>
</tr>
<tr>
<td>(A)</td>
<td>2 m^2</td>
<td>frontal area</td>
</tr>
<tr>
<td>(c_A)</td>
<td>0.3</td>
<td>air resistance coefficient</td>
</tr>
<tr>
<td>(\rho)</td>
<td>1.2 kg/m^3</td>
<td>air density</td>
</tr>
<tr>
<td>(g)</td>
<td>9.81 m/sec^2</td>
<td>gravitational constant</td>
</tr>
<tr>
<td>(c_1)</td>
<td>1.3</td>
<td>empirical coefficient</td>
</tr>
<tr>
<td>(c_2)</td>
<td>10</td>
<td>empirical coefficient</td>
</tr>
<tr>
<td>(c_3)</td>
<td>0.8</td>
<td>empirical coefficient</td>
</tr>
<tr>
<td>(\mu)</td>
<td>1.0</td>
<td>tire/pavement friction coefficient</td>
</tr>
</tbody>
</table>

You might want to load these variables up once from an m-file named `initialize.m`. There are two differential equations here that represent equations of motion, one for the wheel (1) and one for the car (7). If you look through the equations you will see that there appears in them only \(v_C\) and \(\omega\), i.e. \(x_C\) and \(\phi\) do not appear. Nevertheless, we might be interested in how long the car brakes/skids during the braking process. So we will also extract \(x_C\) from the Simulink model.
We start, as normal with the integration backbones, the line of integrals that produce velocity and displacement from acceleration. Produce the models for equations 1 and 7.

These partial models give us $v_C$ and $v_{C,\text{opt}}$, from which we can find $v_{C,\text{opt}}$ and $s$ using equation (2). Then with $s$ we can find $f(s)$ using equation (3). Add these partial models to the diagram.

With $f(s)$ you can calculate $F_B$, using equations (4) and (8). $F_B$ will be the total braking force. To get $F_{B,\text{opt}}$ you simply divide this by 4 for use in the partial model developed for equation (1).

Additionally we want to stop the simulation when the speed gets very low. But we want to stop it before the car speed gets to 0 to avoid a division by 0 in equation (2). You can do this by checking the value of the velocity and stopping the simulation if it drops below 0.01 m/sec.

Set the model up to view the two velocities, $v_C$ and $v_{C,\text{opt}}$ on one scope. Also look at the slip. Look at the braking force on a scope. Look at the distance traveled on a scope too.

**Check the Model**

As the model is set up on a non-ABS car, we apply a steady pedal pressure to produce $M_B$ ($M_B = 4 M_{B,\text{opt}}$) Let’s try the do-nothing case first, where the drivers does not push on the brakes. What happens? What should the velocity do? What should the slip be?

Now apply a light braking load, say 2000 Nm. The car should slow steadily down without slipping.

Now apply a heavy braking load, say 8000. You should see some slip.

What is the difference between the distance to stop with and without slip in the above two cases?

Can you find the optimal $M_B$, so that the car has maximum stopping power without excessive slip? This should correspond to the shortest distance to stop. For what slip value is the stopping power greatest? Through trial and error, find the correct braking to give the shortest stopping distance.

**Car with ABS**

From the diagram on page 3 we can see that the maximum braking coefficient occurs with a slip of about 0.28. Note that this parameter is available to us as modelers, since we calculate s as part of the simulation. Thus we can set up a controller to react if s deviates from a desired value of 0.28. We will use a simple off/on controller for this. If the slip is below 0.28, we want to apply the braking moment. If the slip is above 0.28, we want to release the brakes.

To implement this, form a comparison between the actual $s$ and the desired $s$, $s_d = 0.28$. If $s_d - s < 0$, remove the braking moment. If $s_d - s > 0$, apply the full braking moment. You can implement this with a sign block. Look at this block and see how it works. Follow the sign block with a gain block to provide the proper change in braking moment, say $dM_B/dt = 20,000$ N-m/sec. Then use an integrator, limited to positive values to get $M_B$. 

5
It is interesting now to plot $v_c$ and $v_{c\omega}$ together. Plot also the slip as a function of time and then the car’s position as a function of time. Try this out with various coefficients of frictions ($\mu = 1$ for dry pavement, $\mu = 0.6$ for wet pavement, $\mu = 0.3$ for icy pavement, etc.).