Homework # 6, Math 412 Due Friday, November 13, 2015

This homework set has six (6) problems. Some of them are routine, others require more thought. You are encouraged to work with others and to ask questions of your instructor; however, you must write up your solutions independently. On this and all subsequent homework sets please write neatly and use complete sentences. Writing mathematics well is a craft, aim to hone it!

1. *RAF* 4.3.1

2. Suppose $K$ is compact, $T$ is closed and $K \cap T = \emptyset$. Let

$$d(K, T) = \inf\{|x - y| : x \in K, y \in T\}$$

be the distance from the set $K$ to the set $T$.

(a) Show that $d(K, T) > 0$.

(b) Show that if $K$ is merely closed, $d(K, T) = 0$ is possible.

3. *RAF* 4.3.4

4. *RAF* 4.3.5

5. *RAF* 4.3.8

6. Show that every open interval can be expressed as the countable union of compact sets.

*Challenge*: Let $C$ be the Cantor set. Show that

$$\{x + y : x \in C \text{ and } y \in C\} = [0, 2].$$