16.3-16.4: Double Integrals over General Regions and in Polar Coordinates

These questions are meant to guide your initial study of the material prior to our discussion in class. For the non-computational questions, please give short answers for each: 2-3 sentences or a small paragraph.

Since these questions form an initial investigations of these ideas, you are not expected to always be able to answer these question fully. They are intended to raise key topics and questions that can be discussed in more depth and detail in subsequent classes. Ideally, you should go through the chapter prior to class and try a couple of problems. Then once we’ve had a deeper discussion in class, revisit the chapter (and these questions) and then do many problems covering that material. The reading provides the background which will guide you through the work. The homework problems then provide the experience in implementing the concepts and cover some the variability which can occur in problems for that material.

Caveat: These questions are not intended to be a comprehensive summary of all the ideas in this section; they offer a selected subset of those ideas.

1. Sec 16.3: Defining the Domain.

(a) The regions shown in Figure 5 are all $y$-simple (Type I); which of these regions are also $x$-simple (Type II)?

(b) Which of the $x$-simple regions in Figure 7 are also $y$-simple?

(c) In example 3, the region $D$ is both $x$- and $y$-simple. Considering $D$ as an $x$-simple region implies that $D$ can be defined by

$$D: \{(x,y) \mid c \leq y \leq d, h_1(y) \leq x \leq h_2(y)\}.$$ 

For this example, what are the constant bounds $c$ and $d$ and what are the functions $h_1(y)$ and $h_2(y)$? (See fig. 12b.)

(d) Now reconsider $D$ as a $y$-simple, i.e., $D$ can be defined by

$$D: \{(x,y) \mid a \leq x \leq b, g_1(x) \leq y \leq g_2(x)\}.$$ 

What are the constant bounds $a$ and $b$ and what are the functions $g_1(x)$ and $g_2(x)$? (See fig. 12a.) Note: $g_1(x)$ will be piecewise defined.

2. Sec 16.3: Integral Bounds and Order of Integration.

Consider a double integral over a general region, $\iint_R f(x,y) \, dA$. (You might find example 2 useful for these questions.)

(a) When written as an iterated integral, can the bounds for the outer variable involve either variable or must they both be numbers?
(b) How about for the bounds for the inner variable?

(c) What is $dA$ if the region is $y$-simple (Type I)?

(d) If it’s $x$-simple?

3. **Sec. 16.3: Sketching and Finding the Domain.** Redo example 4 keeping the same tetrahedron except for the edge formed by $y = 2x$; move that edge to another line, $y = kx$ for some $k > 0$ ($k \neq 2$).

4. **Sec. 16.4: Polar Rectangles.** A “polar rectangle” is a region defined by independent intervals (with constant bounds) for the distance from the origin, $r$, and for the angle from the positive $x$-axis, $\theta$; i.e.,

$$D: \{(x, y) \mid a \leq r \leq b, \alpha \leq \theta \leq \beta\}.$$

(a) Sketch a polar rectangle where $0 < \alpha < \pi/2$ and $\pi/2 < \beta < \pi$.

(b) Can the unit disk, the disk bounded by the unit circle, be thought of as a polar rectangle? If so, write down such a definition for the unit disk.

(c) How does the area of a polar rectangle change as the distance from the origin changes? (See Figures 4 and 5.)

(d) What is the relationship between this change in area and the definition of the double integral’s differential $dA$ in polar coordinates? (See Formula 2.)

5. Create a question about this (or any of the preceding) material.