16.7: Triple Integrals in Cylindrical Coordinates

These questions are meant to guide your initial study of the material prior to our discussion in class. For the non-computational questions, please give short answers for each: 2-3 sentences or a small paragraph.

Since these questions form an initial investigations of these ideas, you are not expected to always be able to answer these question fully. They are intended to raise key topics and questions that can be discussed in more depth and detail in subsequent classes. Ideally, you should go through the chapter prior to class and try a couple of problems. Then once we’ve had a deeper discussion in class, revisit the chapter (and these questions) and then do many problems covering that material. The reading provides the background which will guide you through the work. The homework problems then provide the experience in implementing the concepts and cover some the variability which can occur in problems for that material.

Caveat: These questions are not intended to be a comprehensive summary of all the ideas in this section; they offer a selected subset of those ideas.

1. **Coordinate Surfaces:**
   Figure 4 shows an example of a “coordinate surface”, \( r = c \), in cylindrical coordinates, i.e., a constant-\( r \) surface.

   (a) What are the constant-\( \theta \) and constant-\( z \) coordinate surfaces for cylindrical coordinates?

   (b) Sketch and algebraically define a solid which is bounded solely by cylindrical coordinate surfaces”. Hint: the algebraic definitions should have purely numeric bounds for each variable.

2. **Changing Coordinate Systems:**
   Equations 1 and 2 state the coordinate transformations between Cartesian and cylindrical coordinates. Examples 2 and 3 demonstrate how to use them to change the definition of a surface from Cartesian to cylindrical coordinates.

   (a) Find the equation for the hemisphere, \( x^2 + y^2 + z^2 = 4 \), \( z \geq 0 \), in cylindrical coordinates.

   (b) Using Equations 1 and 2, find an equation in Cartesian coordinates for the surface \( r = 2 \cos \theta \). (Hint: multiply by \( r \) first.)

   (c) Sketch the surface in part (b). Hint: move all variables to one side of the equation and complete the square.
3. **Volume Elements and the Differential $dV$:**

Sec 16.7: Figure 7 shows a “volume element” in cylindrical coordinates. Consider a similar-shaped solid with positive, but small, $\Delta r$, $\Delta \theta$ and $\Delta z$.

(a) Does the volume of this solid change (for fixed $\Delta r$, $\Delta \theta$ and $\Delta z$) when it is moved vertically, *i.e.*, a change in $z$ while holding $\theta$ and $r$ fixed?

(b) What about when moving it in $\theta$ holding $r$ and $z$ fixed, *i.e.*, rotation?

(c) What about moving it in $r$ only?

(d) Is this consistent with the definition of the volume element in Formula 2, *i.e.*, $dV = r \, dr \, d\theta \, dz$?

4. **Defining Domains and Iterated Integrals.**

Formula 4 is the iterated triple integral in cylindrical coordinates when the domain $E$ is $z$-simple. Then the two-dimensional projected domain, $D$, is in the $xy$-plane.

Formula 4 then assumes that $D$ can be defined in polar coordinates as $r$-simple: $dV = dz \, dA = r \, dz \, dr \, d\theta$. 3D-domains of this type are easy to describe in cylindrical coordinates, see Examples 3 and 4.

(a) In Example 3 (Figure 8), which surfaces are cylindrical “coordinate surfaces”? Which are not?

(b) What 2D region is $D$, the projection of $E$ into the $xy$-plane? How is $D$ defined in polar coordinates?

(c) What are the bounds for $z$ for each point $(x, y) \in D$?

(d) How are all of these results used to define the iterated integral used in Example 3?

5. Create a question about this (or any of the preceding) material.