8.5: Strategies for Integration

You should know the **basic integration formulas** presented in the Table of Integration Formulas on page 542 of your textbook. You do not need to know formulas 15 or 16. You should be able to derive Formulas 13 and 14 using $u$-substitution. You should be able to derive Formulas 17, 18, and 20 using trigonometric substitutions. You should be able to derive Formula 19 using partial fractions. In addition to knowing these basic formulas, you should know the following **strategies for integration**:

- **Simplify the integrand.** Use algebraic manipulation or trigonometric identities to try to simplify the integrand.

- **$u$-Substitution.** Look for a helpful $u$-substitution. If there is an expression of the form $\sqrt{g(x)}$ in the integrand, you can try the substitution let $u = \sqrt{g(x)}$.

- **Trigonometric functions.** If the integrand is a product of powers of $\sin x$ and $\cos x$, or a product of powers of $\tan x$ and $\sec x$, or a product of powers of $\cot x$ and $\csc x$, try to use the trigonometric identities and substitution to evaluate the integral.

- **Trigonometric substitution.** If the integrand contains a factor of the form $\sqrt{a^2 - x^2}$ or $a^2 - x^2$; $\sqrt{a^2 + x^2}$ or $a^2 + x^2$; or $\sqrt{x^2 - a^2}$ or $x^2 - a^2$; a trigonometric substitution can often be helpful. You can draw a right triangle to determine which trigonometric substitution to make.

- **Partial fractions.** If the integrand is a rational function (polynomial over a polynomial), use partial fractions.

- **Integration by parts.** If the integrand contains a product of a power of $x$ (or a polynomial) and a transcendental function (such as a trigonometric, exponential, or logarithmic function), try IBP: $\int u \, dv = uv - \int v \, du$.

- You may need to use several methods to evaluate a given integral.

**Problems.** Evaluate each of the following integrals.

1. $\int \frac{\tan^3 x}{\cos^3 x} \, dx$
2. $\int \cos(\sqrt{x}) \, dx$
3. $\int \frac{1}{x \sqrt{\ln x}} \, dx$
4. $\int \frac{x}{x^4 + 10x^2 + 9} \, dx$
5. $\int \frac{1-x}{1+x} \, dx$
6. $\int \frac{1}{e^{3x} - e^x} \, dx$
7. $\int \frac{e^{2x}}{1 + e^{4x}} \, dx$
8. $\int x \cos^{-1} x \, dx$
9. $\int \frac{\cos x}{\sqrt{\sin^2 x - 25}} \, dx$
10. $\int \frac{1}{1 + 2e^x - e^{-x}} \, dx$
11. $\int 3^x \cos(3^x) \sin(3^x) \, dx$
12. $\int \ln(x^2) \, dx$