Note: I reversed problem 1 class as inside both circles, a harder problem.

\[ x^2 + (y-1)^2 = 1 \iff x^2 + y^2 = 2y \]

\[ x^2 + y^2 = 1 \]

There is neither a type I nor type II in cartesian, so let's try polar.

\[ x^2 + y^2 = 1 \iff r^2 = 1 \quad \text{or} \quad r = 1 \]

\[ x^2 + y^2 = 2y \implies r^2 = 2r \sin \theta \implies r = \frac{2 \sin \theta}{0 \leq \theta \leq \pi} \]

This is a good type II polar graph.

\[ \Omega : \{ (r, \theta) \mid \theta_1 \leq \theta \leq \theta_2, g_1(\theta) \leq r \leq g_2(\theta) \} \]

We need both \( g_1(\theta) \) and \( g_2(\theta) \), but we've already got \( g_1(\theta) = 1 \) since \( x^2 + y^2 = 1 \implies r = 1 \).

\[ g_2(\theta) = 2 \sin \theta \quad \text{and} \quad x^2 + y^2 = 1 \implies r = 2 \sin \theta \]

Still need to find \( \theta_1 \) and \( \theta_2 \).

Just set \( g_1(\theta) = g_2(\theta) \implies 2 \sin \theta = 1 \implies \sin \theta = \frac{1}{2} \)

\[ \theta = \frac{\pi}{6} \quad \text{or} \quad \theta = \frac{5\pi}{6} \]

\[ \frac{1}{2} \quad \text{or} \quad \frac{1}{2} \]

So \( \Omega : \frac{\pi}{6} < \theta < \frac{5\pi}{6}, \quad 1 < r < 2 \sin \theta \).

Now for density in polar, we get \( \rho = \frac{b}{r} \), note we're far from the origin in \( \Omega \), so \( r \neq 0 \).

So the mass integral becomes

\[ \int \int_{\Omega} \rho(x,y) \, dA = \int_{\pi/6}^{5\pi/6} \int_{1}^{2 \sin \theta} \frac{b}{r} \, r \, dr \, d\theta \]

\[ = \int_{\pi/6}^{5\pi/6} \frac{b}{2} \, d\theta = \frac{b\pi}{6} \]
\[
m = k \int_{\pi/6}^{5\pi/6} \int_0^{2\pi} dr \, d\theta = k \int_{\pi/6}^{5\pi/6} (2\sin \theta - 1) \, d\theta
\]

\[= \left[-2\cos \theta - \theta \right]_{\pi/6}^{5\pi/6} = k \left[2\sqrt{3} - \frac{5\pi}{3} \right]\]

\[
m_c = \frac{m}{k} \times 1.37 \Rightarrow k > 0 \quad \text{mass is positive}
\]

From symmetry: \( M_y = \oint y \, g(y) \, dV = 0 \)

Since lambda is symmetric in both shape and density about \( y = 0 \)

\[\int_{\pi/6}^{5\pi/6} \left( \cos \theta + \frac{1}{r} \right) r \, dr \, d\theta = 0 \]

\[
M_x = \oint y \, g(y) \, dV = \int_{\pi/6}^{5\pi/6} \left( \frac{1}{r} + \cos \theta \right) r \, dr \, d\theta
\]

\[= \frac{k}{2} \int_{\pi/6}^{5\pi/6} 2\sin \theta \, d\theta = \frac{k}{2} \int_{\pi/6}^{5\pi/6} \frac{1}{2} \sin \theta \, d\theta
\]

\[= \frac{k}{2} \left[ \frac{1}{3} \cos \theta + \cos \theta \right]_{\pi/6}^{5\pi/6}
\]

\[= \frac{k}{2} \left[ \frac{1}{3} \frac{\sqrt{3}}{2} - 3 \frac{\sqrt{3}}{2} \right]
\]

\[= \sqrt{3} \frac{k}{2}
\]

\[\bar{x} = \frac{M_y}{m} = \frac{0}{\frac{3\sqrt{3}}{2}} = \frac{\sqrt{3} \frac{k}{2}}{\frac{3\sqrt{3}}{2}} - \frac{5\pi}{3} \approx 1.26
\]