1. Number 11, page 68 AM.

2. Let $K \subset L$ be fields with $K$ algebraically closed. Let $f_1, \ldots, f_m$ be polynomials in $x_1, \ldots, x_n$ which vanish simultaneously at a point in $L^n$. Show that they vanish simultaneously at some point of $K^n$.

3. Let $K \subset L$ be fields with $K$ algebraically closed. Let $F$ be an irreducible polynomial in $K[z_1, \ldots, z_n]$, a polynomial ring. Show that $F$ is irreducible in $L[z_1, \ldots, z_n]$. (It may help to think of the problem of factoring $F$ as a problem in solving equations).

4. Let $A \subset B$ be rings and suppose that $A$ is a direct summand of $B$ an an $A$-module (i.e. $B = A \oplus_A E$ where $E$ is an $A$ submodule but need not be an ideal). Prove that if $B$ is Noetherian, the $A$ is Noetherian. Hint: use a result we proved in class.

5. If $A$ is a commutative ring, define a topology on Spec $A$ by letting a set of prime ideals be closed if and only if it has the form $V(W)$ for some subset $W$ of $A$, where $V(W) = \{ P \in \text{Spec } A \mid P \supset W \}$.

   (a) Show that if $I$ is the ideal generated by $W$ and $J$ is the radical of $I$, then $V(W) = V(J)$.

   (b) Verify that the sets $V(W)$ satisfy the axioms for the closed sets of a topology on Spec $A$.

   (c) Verify that there is a bijection between the closed sets in Spec $A$ and the radical ideals of $A$ given by letting $J$ correspond to $V(J)$.

    Reminder: a topology on a set $X$ is a set $T$ of subsets of $X$ called closed sets such that
    i. $\emptyset \in T$,
    ii. $X \in T$, and
    iii. $T$ is closed under infinite intersections and finite unions.

6. Suppose $A \subset B$ are rings and that $B$ is module finite as an $A$-algebra. Let $p$ be a prime ideal of $A$. The lying over theorem shows that there is at least one prime $q$ of $B$ with $q \cap A = p$. Show that there are only finitely many such primes $q$. Hint: Suppose $B$ can be generated as an $A$-module by $n$ elements, and suppose there were $n+1$ such $q$, say $q_1, \ldots, q_{n+1}$. Localize both $A$ and $B$ at $S = A - p$, let $m$ be the extension of $p$ to $A_p$, and let $q'_i$ be the extension of $q_i$ to $S^{-1}B$. What kind of ideals are the $q'_i$? What kind of ring extensions do you get if you divide $A_p$ by $m$ and $S^{-1}B$ by $\cap q'_i$?

7. Show by example that the preceding problem would be false if we assumed only that $B$ were integral over $A$.

8. Let $D$ be a domain and let $\{ D_i \}_{i \in I}$ be a family of sub-domains of $D$, for some index set $I$. Suppose each $D_i$ is normal. Show that $\cap_{i \in I} D_i$ is normal.