1. Suppose the Barber of Seville is a man who lives in Seville. Decide if the following sentence is a proposition:
The Barber of Seville shaves those men (and only those men) in the town of Seville who do not shave themselves.

2. Construct truth tables for the following propositional forms:
   (a.) $P \lor (Q \lor R)$
   (b.) $(P \lor Q) \lor R$

   Note that these two truth tables are the same (assuming you listed the inputs in the same order). This means that we could replace one form with the other and always get the same truth value when we replaced the letters with actual propositions. Because of this we say that $\lor$ is associative and will write $P \lor Q \lor R$ to mean $(P \lor Q) \lor R$.

   In general, when two propositional forms have the same truth tables we say they are equivalent. We use the symbol $\iff$ to denote equivalence: so in the previous problem: $P \lor (Q \lor R) \iff (P \lor Q) \lor R$. It is also true that $P \land (Q \land R) \iff (P \land Q) \land R$, and we hold to a similar notational convention as for $\lor$.

3. Construct truth tables for the following propositional forms:
   (a.) $\neg P \lor Q$
   (b.) $\neg (P \lor Q)$
   (c.) $(\neg P) \land (\neg Q)$

   The convention is that negation always modifies the closest letter, so $\neg P \lor Q$ means $(\neg P) \lor Q$ instead of $\neg (P \lor Q)$.

4. Demonstrate that $P \lor (Q \land R) \iff (P \lor Q) \land (P \lor R)$. Sometimes we say that $\lor$ distributes over $\land$.

   Rules like the one in the previous problem can help us determine equivalences without writing truth tables once we realize that equivalence is a transitive property. That is to say, if you have three propositional forms, say $F_1$, $F_2$, and $F_3$ (maybe $F_1$ is the form $P \lor (\neg Q) \lor (R)$, just by way of an example), then if $F_1$ and $F_2$ have the same truth table, and $F_2$ and $F_3$ have the same truth table, then $F_1$ and $F_3$ have the same truth table. Said more concisely, if $F_1 \iff F_2$ and $F_2 \iff F_3$, then $F_1 \iff F_3$.

   To give a concrete example, recall that we showed in class that $\neg (P \lor Q) \iff (\neg P) \land (\neg Q)$. It is also true that $\neg (P \land Q) \iff (\neg P) \lor (\neg Q)$. So

   $$\neg (P \lor (Q \land R)) \iff \neg P \land \neg (Q \land R) \iff \neg P \land (\neg Q \lor \neg R) \iff \neg P \land (\neg Q) \lor (\neg P \land \neg R),$$

   that is,

   $$\neg (P \lor (Q \land R)) \iff (\neg P \land \neg Q) \lor (\neg P \land \neg R).$$

5. Demonstrate that $\neg (P \lor Q \land R) \iff \neg P \land \neg Q \land \neg R$ without using truth tables (as above).

6. Show that the propositional form $(P \implies Q) \land P \implies Q$ is a tautology (that is, it is true for every possible proposition $P$ and $Q$. This means that no matter what $P$ and $Q$ may be, you may suppose that $(P \implies Q) \land P \implies Q$ is true.

   Sometimes translations from English to Mathspeak and back can require a bit of care.
7. Rephrase the following statements in the form “If $P$ then $Q$.”

(a) Either I’m dreaming or a giant crocodille just handed me a ten dollar bill.
(b) Only fools fall in love.

8. Let $P$, $M$, and $J$ be the statements that Paul, Mike and John ate lunch today. Using symbols $P$, $Q$, $R$, $\implies$, $\neg$, etc, translate the following into Mathspeak.

(a) Paul ate lunch today, but Mike did not.
(b) Paul and Mike either both ate lunch, or both did not eat lunch.
(c) Neither Mike nor John ate lunch today.
(d) If Mike did not eat lunch, then Paul ate lunch but John didn’t each lunch.
(e) Precisely one of Mike and Paul ate lunch today.

The grader will carefully consider 5 and 7.