Each cash flow of a fixed income security must be discounted at the factor or rate appropriate for the term of that cash flow.

<table>
<thead>
<tr>
<th>Time to Maturity</th>
<th>Discount Factor</th>
<th>Spot Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.97557</td>
<td>5.008%</td>
</tr>
<tr>
<td>1</td>
<td>0.95247</td>
<td>4.929%</td>
</tr>
<tr>
<td>1.5</td>
<td>0.93045</td>
<td>4.864%</td>
</tr>
<tr>
<td>2</td>
<td>0.90796</td>
<td>4.886%</td>
</tr>
<tr>
<td>2.5</td>
<td>0.88630</td>
<td>4.887%</td>
</tr>
</tbody>
</table>
Yield-to-Maturity

- Yield-to-maturity, or yield, is the single rate that when used to discount a bond's cash flows, produces the bond's market price.
- Yield is not a good measure of relative value or of realized return to maturity.
- If a bond's YTM remains unchanged over a short time period, that bond's realized total rate of return equals its yield.

1.5 year maturity, coupon rate is 4.5%, price is 105.856

\[
105.856 = \frac{2.25}{1 + \frac{y}{2}} + \frac{2.25}{\left(1 + \frac{y}{2}\right)^2} + \frac{2.25}{\left(1 + \frac{y}{2}\right)^3}
\]

The price of a T-year security making semiannual payments of \(c/2\) and a final principal payment of \(F\) is:

\[
P(T) = \frac{c}{y} \left[1 - \left(\frac{1}{1 + y/2}\right)^{2T}\right] + F \left(\frac{1}{1 + y/2}\right)^{2T}
\]
From the formula

- When Coupon Rate = YTM. Par bonds.
- When Coupon Rate > YTM, Premium bonds. The bonds are selling for more than face value.
- When Coupon Rate < YTM, Discount bonds, or the bond sells at a discount to par.

Bond prices approach par as they approach maturity: pull to par.

If a bond’s YTM over a six-month period remains unchanged, the annual total return of the bond over that period equals its YTM.

\[
P_0 = \frac{c}{1 + \frac{y}{2}} + \frac{c}{(1 + \frac{y}{2})^2} + \cdots + \frac{1 + c/2}{(1 + \frac{y}{2})^{2T}}
\]

\[
P_{1/2} = \frac{c}{2} + \frac{c}{1 + \frac{y}{2}} + \cdots + \frac{1 + c/2}{(1 + \frac{y}{2})^{2T-1}}
\]

\[
P_{1/2} = (1 + \frac{y}{2})P_0
\]

\[
y = 2\left(\frac{P_{1/2}}{P_0} - 1\right)
\]
6.25s, 2 year maturity, \( YTM = ?? \)

Spot Rates: 5.008\%, 4.929\%, 4.864\%, 4.886\%

\[
102 + \frac{18.125}{32} = \frac{3.125}{1 + \frac{y}{2}} + \left(1 + \frac{y}{2}\right)^{-1} \cdot \frac{3.125}{1 + \frac{y}{2}} + \left(1 + \frac{y}{2}\right)^{-2} \cdot \frac{3.125}{1 + \frac{y}{2}} + \left(1 + \frac{y}{2}\right)^{-3} \cdot \frac{103.125}{1 + \frac{y}{2}}
\]

- Flat term structure of spot rates, \( YTM = \text{Spot} \)
- Upward sloping, \( YTM \) is below the last spot
- Downward sloping, \( YTM \) above last spot

**Yield, cash flow, and spot rates**

- Yield of a zero coupon bond of a particular maturity equals the spot rate of that maturity.
- Par coupon bonds: coupon bonds selling at par, the yield equals the coupon rate \( c \).
Par Coupon Bonds

\[ \frac{100c}{2} \sum_{t=1}^{2T} d(t/2) + 100d(T) = 100 \]
\[ c = \frac{2[1-d(T)]}{\sum_{t=1}^{2T} d(t/2)} \]

Par Nonprepayable Mortgage:

present value=amount borrowed=par
yield discounts future cash flows into PV

\[ X \sum_{t=1}^{2T} d(t/2) = 100 \]
\[ X = 100 / \left( \sum_{t=1}^{2T} d(t/2) \right) \]
\[ 100 = X \sum_{t=1}^{2T} \frac{1}{(1+y_T/2)^t} \]

Discussion of Figure

- All equal at .5 years
- Short-end, downward sloping, par yields exceed zero yields, negligible
- Medium-term, upward sloping, zero yields exceed par yields, spread increasing
- Long-end, down sloping, spread narrows
- Qualitatively, relative to zero yield, mortgage yield is just like par yield, except more pronounced
**YTM and relative value: the coupon effect**

- The impact of coupon level on the YTM of coupon bonds with the same maturity is called the coupon effect.
- The size of the coupon effect depends very much on the shape of the term structure of interest rates and the cash flow structure of the securities.

**YTM and realized return**

\[
\frac{102 + 18.125}{32} = \frac{3.125}{1 + \frac{Y}{2}} + \frac{3.125}{1 + \frac{Y}{2}} + \frac{3.125}{1 + \frac{Y}{2}} + \frac{103.125}{1 + \frac{Y}{2}}
\]

\[
102.5664\left(1 + \frac{Y}{2}\right)^4 = 3.125\left[\left(1 + \frac{Y}{2}\right)^4 + \left(1 + \frac{Y}{2}\right)^4 + \left(1 + \frac{Y}{2}\right)^4 + \left(1 + \frac{Y}{2}\right)^4\right] + 103.125
\]

- YTM is not the bond's return if held to maturity
- Coupons are invested at uncertain future rates