2.1 You invest $100 for two years at 2% compounded semiannually. How much do you have at the end of the two years?

A. Since two years are 4 semiannual periods,:

\[
100 \left( 1 + \frac{2\%}{2} \right)^4 = 104.0604
\]

2.2 You invested $100 for three years and, at the end of those three years, your investment was worth $107. What was your semiannually compounded rate of return?

A. Solve for the rate \( r \) in the equation

\[
100 \left( 1 + \frac{r}{2} \right)^6 = 107
\]

\[r = 2.2681\%\]

2.3 Using the discount factors in the table, derive the corresponding spot and forward rates.

<table>
<thead>
<tr>
<th>TERM</th>
<th>DISCOUNT FACTOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>.5</td>
<td>.998752</td>
</tr>
<tr>
<td>1</td>
<td>.996758</td>
</tr>
<tr>
<td>1.5</td>
<td>.993529</td>
</tr>
</tbody>
</table>

A. Solve the following equations for the forward rates

\[
.998752 = \frac{1}{1 + \frac{f(1.5)}{2}}
\]

\[
.996758 = \frac{.998752}{1 + \frac{f(1)}{2}}
\]

\[
.993529 = \frac{.996758}{1 + \frac{f(1.5)}{2}}
\]

to obtain \( f(.5) = .25\% \); \( f(1) = .4\% \); \( f(1.5) = .65\% \).
Then solve the following equations for the spot rates

\[ \hat{r}(.5) = f(.5) \]

\[
\left(1 + \frac{\hat{r}(1)}{2}\right)^2 = \left(1 + \frac{\hat{r}(.5)}{2}\right) \left(1 + \frac{f(1)}{2}\right)
\]

\[
\left(1 + \frac{\hat{r}(1.5)}{2}\right)^3 = \left(1 + \frac{\hat{r}(1)}{2}\right)^2 \left(1 + \frac{f(1.5)}{2}\right)
\]

to get \( \hat{r}(.5) = .25\% \); \( \hat{r}(1) = .3250\% \); \( \hat{r}(1.5) = .4333\% \).

Note that there are other mathematically equivalent ways to obtain these same results.