MILITARY DECISION AND GAME THEORY*

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The United States military doctrine of decision prescribes that a commander select the course of action which offers the greatest promise of success in view of the enemy’s capabilities of opposing him. This paper analyzes two battle decisions of World War II, and develops the analogy between existing military doctrine and the ‘theory of games’ proposed by von Neumann. Current U. S. doctrine is conservative. The techniques of game theory permit analysis of the risk involved if the commander deviates from current doctrine to base his decision on his estimate of what his enemy intends to do rather than on what his enemy is capable of doing. The idea of ‘mixed strategies’ presents more difficulties but may be useful, particularly for command decisions for small military organizations.

VON NEUMANN and Morgenstern point out[1] that in the early stages of the development of a new theory, application serves to corroborate theory. The theory of games has been most fully developed for the two-person situation, the conflict of two opposing individuals or groups. Almost all battle decisions involve two opposing military forces. Moreover, the student of game theory need not analyze numerous battles to learn the military philosophy of decision. The doctrine has been formalized and is available in military texts.2

MILITARY-DECISION DOCTRINE

A military commander may approach decision with either of two philosophies. He may select his course of action on the basis of his estimate of what his enemy is able to do to oppose him. Or, he may make his selection on the basis of his estimate of what his enemy is going to do. The former is a doctrine of decision based on enemy capabilities; the latter, on enemy intentions.

The doctrine of decision of the armed forces of the United States is a doctrine based on enemy capabilities. A commander is enjoined to select

*The basic concepts of this paper were developed by the author, then a colonel in the U. S. Air Force, while a student at the Air War College, 1940–1950.
the course of action which offers the greatest promise of success in view of the enemy capabilities. The process of decision, as approved by the Joint Chiefs of Staff and taught in all service schools, is formalized in a five-step analysis called the Estimate of the Situation. These steps are illustrated in the following analysis of an actual World War II battle situation.

Step 1. The Mission

General MacArthur as Supreme Commander had ordered Kenney to intercept and inflict maximum destruction on the convoy. This then was Kenney's mission.

Step 2. Situation and Courses of Action

The situation as outlined above was generally known. One new critical factor was pointed out by Kenney's staff. Rain and poor visibility were predicted for the area north of New Britain. Visibility south of the island would be good.

The Japanese commander had two choices for routing his convoy from Rabaul to Lae. He could sail north of New Britain, or he could go south of that island. Either route required three days.

Kenney considered two courses of action, as he discusses in his memoirs. He could concentrate most of his reconnaissance aircraft either along the northern route where visibility would be poor, or along the southern route where clear weather was predicted. Mobility being one of the great advantages of air power, his bombing force could strike the convoy on either route once it was spotted.

Step 3. Analysis of the Opposing Courses of Action

With each commander having two alternative courses of action, four possible conflicts could ensue. These conflicts are pictured in Fig. 2.

Step 4. Comparison of Available Courses of Action

If Kenney concentrated on the northern route, he ensured one of the two battles of the top row of sketches. However, he alone could not determine which one of these two battles in the top row would result from his decision. Similarly, if Kenney concentrated on the southern route, he ensured one of the battles of the lower row. In the same manner, the Japanese commander could not select a particular battle, but could by his decision assure that the battle would be one of those pictured in the left column or one of those in the right column.

Kenney sought a battle which would provide the maximum opportunity for bombing the convoy. The Japanese commander desired the minimum exposure to bombing. But neither commander could determine the battle which would result from his own decision. Each commander had full and independent freedom to select either one of his alternative strategies. He had to do so with full realization of his opponent's freedom of choice. The particular battle which resulted would be determined by the two independent decisions.
The U. S. doctrine of decision—the doctrine that a commander base his action on his estimate of what the enemy is capable of doing to oppose him—dictated that Kenney select the course of action which offered the greatest promise of success in view of all of the enemy capabilities. If Kenney concentrated his reconnaissance on the northern route, he could expect two days of bombing regardless of his enemy's decision. If Kenney selected his other strategy, he must accept the possibility of a less favorable outcome.

**Step 5. The Decision**

Kenney concentrated his reconnaissance aircraft on the northern route.

**Discussion**

Let us assume that the Japanese commander used a similar philosophy of decision, basing his decision on his enemy's capabilities. Considering the four battles as sketched, the Japanese commander could select either the left or the right column, but could not select the row. If he sailed the northern route, he exposed the convoy to a maximum of two days of bombing. If he sailed the southern route, the convoy might be subjected to three days of bombing. Since he sought minimum exposure to bombing, he should select the northern route.

These two independent choices were the actual decisions which led to the conflict known in history as the Battle of the Bismarck Sea. Kenney concentrated his reconnaissance on the northern route; the Japanese convoy sailed the northern route; the convoy was sighted approximately one day after it sailed; and Allied bombing started shortly thereafter. Although the Battle of the Bismarck Sea ended in a disastrous defeat for the Japanese, we cannot say the Japanese commander erred in his decision. A similar convoy had reached Lae with minor losses two months earlier. The need was critical, and the Japanese were prepared to pay a high price. They did not know that Kenney had modified a number of his aircraft for low-level bombing and had perfected a deadly technique. The U. S. victory was the result of careful planning, thorough training, resolute execution, and tactical surprise of a new weapon—not of error in the Japanese decision.

Those familiar with game theory will recognize that the Rabaul-Lae situation presents all the features of a two-person game. The two commanders have independent choices of action, and these interact to determine a particular battle. The Bismarck-Sea battle exposed the Japanese convoy to a certain number of days of bombing. The 'game' situation

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**Fig. 2. Possible Battles for the Rabaul-Lae Convoy Situation.** Four different engagements of forces may result from the interaction of Kenney's two strategies with the two Japanese strategies. Neither commander alone can determine which particular battle will result.
may be presented quite simply in a table or matrix. The Kenney strategies are the rows; the Japanese strategies are the columns; at the intersection of each row and column we write the number of days of bombing for such a clash of strategies:

<table>
<thead>
<tr>
<th>Kenney strategies</th>
<th>Japanese strategies</th>
<th>#1-Northern route</th>
<th>#2-Southern route</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1-Northern route</td>
<td>2 days</td>
<td>2 days</td>
<td></td>
</tr>
<tr>
<td>#2-Southern route</td>
<td>1 day</td>
<td>3 days</td>
<td></td>
</tr>
</tbody>
</table>

This matrix shows, with numbers, exactly the same information as has been shown previously with the four battle sketches.

To solve this problem, game theory calls for an additional row and an additional column. Kenney, the maximizing player, had independent choice of the row. He could assure himself an outcome not less than the minimum in any row. Putting the minimum values of each row in a new column at the right, Kenney could look at this column of minimums and select the maximum—the maximum of the minimums, or the *maximin*.

<table>
<thead>
<tr>
<th>Kenney strategies</th>
<th>Japanese strategies</th>
<th>Minimum of row</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>2 days</td>
<td>2 days (maximin)</td>
</tr>
<tr>
<td>#2</td>
<td>1 day</td>
<td>1 day</td>
</tr>
<tr>
<td>Maximum of column</td>
<td>2 days (minimax)</td>
<td>3 days</td>
</tr>
</tbody>
</table>

Similarly, the Japanese commander wanted to minimize his exposure to bombing. He could look at each column and note the worst that could happen to him, the maximum value in the column. These are recorded in the added row—the maximum of each column. For the Japanese commander to assure the minimum exposure to bombing, he could look at these maximums of the columns and select the minimum—the *minimax*.

The treatment of the Rabaul-Lae convoy situation under the military doctrine has obvious similarity to game theory treatment of the same situation. The similarity is not mere coincidence. In fact, as was first noted by the author, the doctrine of the Estimate of the Situation is identical with one of von Neumann's solutions of the two-person zero-sum game (the minorant game) and is based on the same philosophy. A military commander not previously exposed to game theory would probably overlook the convenience of ordering the opposing strategies into rows and columns. However, whether a commander recognizes it or not, the fact remains that the doctrine is identical to a particular solution of game theory. Conclusions derived with mathematical exactness in the theory of games may be applied with equal validity to the military doctrine. Thus the theory permits a more penetrating study of the military doctrine.

Before proceeding further it may be well to clarify that the game theory solution as presented applies only to one particular game—the two-person zero-sum game. By two-person, we mean the conflict of two opposing individuals or groups. By zero-sum, we mean that payments at the end of the conflict are made only among the players. Poker is a zero-sum game, since one player wins only what another loses. A battle may be considered zero-sum if an outcome judged good by one commander is judged equally bad by the other. This assumption has been made implicitly in the analysis of the Battle of the Bismarck Sea. For this situation it is a good assumption.

There is a significant feature in the Rabaul-Lae convoy situation. Had either commander discovered his opponent's decision, he would have gained nothing by changing his own. For this situation there existed 'matched strategies.' The good strategy for Kenney remained good even if the Japanese commander had discovered Kenney's intended strategy and the good strategy for the Japanese remained good even if Kenney discovered it.

There are some parlor games which have matched strategies. For such parlor games, for example ticktacktoe, there is really no point in playing the game. In ticktacktoe, the first player may play so as to assure he wins or draws. The second player may also play so as to assure that he either wins or draws. Between good players only one outcome is possible, a draw. Either player may secure a better outcome only if his opponent makes a mistake; that is, if he plays a strategy that is not good.

Ticktacktoe is a very simple game. More complex games may also have matched strategies. Von Neumann proves that matched strategies must exist for the game of chess, a highly complicated game with thousands of different strategies of play being available to each player. Between perfect players, only one outcome is possible for a game of chess. Fortunately for chess enthusiasts, the game is so complex that these matched strategies have never been discovered.

**THE AVRANCHES-GAP SITUATION**

More often, for both parlor games and battles, there are no matched strategies and one may profit from learning an opponent's decision. A