

# Cal Poly Department of Mathematics

## Puzzle of the Week

Oct 14 - 26, 2016

Let  $N > 1$  be a square integer. Find the integer part of

$$1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \dots + \frac{1}{\sqrt{N}}.$$

*Solutions should be submitted to Morgan Sherman:*

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*before the due date above. Those with correct and complete solutions will have their names listed on the puzzle's web site (see below) as well as in the next email announcement. Anybody associated to Cal Poly is welcome to make a submission.*

<http://www.calpoly.edu/~sherman1/puzzleoftheweek>

*Solution:* The sum has integer part  $2\sqrt{N} - 2$ .

Let  $s$  denote the sum. By comparing the sum with related integrals we find the inequalities:

$$\int_1^{N+1} \frac{dx}{\sqrt{x}} < s < 1 + \int_1^N \frac{dx}{\sqrt{x}}.$$

Calculating these integrals we find that

$$2\sqrt{N+1} - 2 < s < 2\sqrt{N} - 1$$

Hence

$$2\sqrt{N} - 2 < s < 2\sqrt{N} - 1$$

and the result follows.