

Cal Poly Department of Mathematics

Puzzle of the Week

May 19 - May 31, 2016

Consider the ellipse $x^2/a^2 + y^2/b^2 = 1$. It is well known that there is no “simple” formula for length L of its perimeter. However one might approximate the length with either of the formulas:

$$L \approx \pi(a + b), \quad \text{or} \quad L \approx 2\pi\sqrt{ab}$$

Both of these approximations are exact when $b = a$. In this problem we investigate which is a *better* approximation when b is very close to (but not equal to) a .

To do this we choose units so that $a = 1$, and let L denote the length of the perimeter of the ellipse $x^2 + y^2/b^2 = 1$. Let $b = 1 + \epsilon$ and calculate

$$L - \pi(1 + b) \quad \text{and} \quad L - 2\pi\sqrt{b}$$

correct to second order in ϵ . Use this to determine which is the better approximation, and by what approximate factor it is better.

Solutions should be submitted to Morgan Sherman:

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before the due date above. Those with correct and complete solutions will have their names listed on the puzzle's web site (see below) as well as in the next email announcement. Anybody associated to Cal Poly is welcome to make a submission.

<http://www.calpoly.edu/~sherman1/puzzleoftheweek>