Cal Poly Department of Mathematics

Puzzle of the Week

Jan 16 - 28, 2015

Kent Morrison relayed the following problem, which originated with David Farmer:

Calculate the sum of the infinite series:

$$.1 + .011 + .00111 + .0001111 + .000011111 + \dots$$

whose nth term has, following the decimal point, n ones preceded by $n-1$ zeros.

Solutions should be submitted to Morgan Sherman:

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before next Thursday. Those with correct and complete solutions will have their names listed on the puzzle's web site (see below) as well as in next week's email announcement. Anybody is welcome to make a submission.

http://www.calpoly.edu/~sherman1/puzzleoftheweek

Solution: The series sums to $\frac{100}{891} = 0.112233445566778899001122\dots$

First the nth term can be expressed as

$$0.\underbrace{0\cdots0}_{n-1}\underbrace{1\cdots1}_{n} = \frac{\frac{1}{9}(10^n-1)}{10^{2n-1}}$$

So then the sum is:

$$\begin{aligned}\frac{1}{9}\sum_{n=1}^{\infty}\frac{10^n-1}{10^{2n-1}} &= \frac{1}{9}\left(\sum_{n=1}^{\infty}\frac{1}{10^{n-1}} - \frac{1}{10}\sum_{n=1}^{\infty}\frac{1}{(10^2)^n}\right)\\ &= \frac{1}{9}\left(\frac{1}{1-\frac{1}{10}} - \frac{1}{10}\frac{1}{1-\frac{1}{100}}\right)\\ &= \frac{1}{9}\left(\frac{10}{9} - \frac{10}{99}\right) = \frac{100}{9\cdot 99}\end{aligned}$$