## Cal Poly Department of Mathematics

# Puzzle of the Week

Nov 20 - Dec 1, 2014

From Tom O'Neil:

A triangle has vertices $A, B, C$, with right angle at vertex $A$. Suppose $D$ is a point on the hypotenuse $\overline{BC}$ such that $|BD| = |AD| + 1$, $\angle BAD = \pi/4$ and $|DC| = 1$. Calculate the length $|AD|$.

*Solutions should be submitted to Morgan Sherman:*

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*before next Thursday. Those with correct and complete solutions will have their names listed on the puzzle's web site (see below) as well as in next week's email announcement. Anybody is welcome to make a submission.*

http://www.calpoly.edu/~sherman1/puzzleoftheweek

Solution: The length of $\overline{AD}$ is $\dfrac{\sqrt{2\sqrt{3}} + \sqrt{3} - 1}{2}$.

Let $\theta$ denote the angle $\angle ABD$ and let $x$ denote the length $|AD|$. Applying the Law of Sines to the triangle $ABD$ we find

$$\frac{\sin\left(\frac{\pi}{2} - \theta\right)}{x} = \frac{\sin\frac{\pi}{4}}{1} \quad \Longrightarrow \quad x = \sqrt{2}\cos\theta. \tag{1}$$

While applying the same law to triangle $ADC$ we find

$$\frac{\sin\theta}{x} = \frac{\sin\frac{\pi}{4}}{x+1} \quad \Longrightarrow \quad \frac{x}{x+1} = \sqrt{2}\sin\theta. \tag{2}$$

Combining (1) and (2) using $\cos^2\theta + \sin^2\theta = 1$ we end up with the equation

$$\left(\frac{x}{x+1}\right)^2 + x^2 = 2, \quad \text{or} \quad x^2 + x^2(x+1)^2 = 2(x+1)^2 \tag{3}$$

which is a quartic equation in $x$. I used *sage* to compute the roots, two of which are imaginary, and of the two real roots the only one which is positive is the answer given above.