

Cal Poly Department of Mathematics

Puzzle of the Week

Oct 30 - Nov 5, 2014

Compute, with justification, the value of

$$\lim_{x \rightarrow \infty} x \int_0^x e^{t^2 - x^2} dt$$

Solutions should be submitted to Morgan Sherman:

*Dept. of Mathematics, Cal Poly
Email: sherman1 -AT- calpoly.edu
Office: bldg 25 room 310*

before next Thursday. Those with correct and complete solutions will have their names listed on the puzzle's web site (see below) as well as in next week's email announcement. Anybody is welcome to make a submission.

<http://www.calpoly.edu/~sherman1/puzzleoftheweek>

Solution: The limit is $\frac{1}{2}$.

We apply the Fundamental Theorem of Calculus, as well as L'Hôpital's Rule:

$$\begin{aligned} \lim_{x \rightarrow \infty} x \int_0^x e^{t^2 - x^2} dt &= \lim_{x \rightarrow \infty} \frac{\int_0^x e^{t^2} dt}{\frac{e^{x^2}}{x}} \\ &= \lim_{x \rightarrow \infty} \frac{e^{x^2}}{-\frac{e^{x^2}}{x^2} + \frac{2xe^{x^2}}{x}} \\ &= \lim_{x \rightarrow \infty} \frac{1}{-\frac{1}{x^2} + 2} \\ &= \frac{1}{0 + 2} = \frac{1}{2} \end{aligned}$$