

Cal Poly Department of Mathematics

Puzzle of the Week

Oct 23 - 29, 2014

A follow-up to last week's problem suggested by Kent Morrison:

A (somewhat smaller than last week's) book has pages numbered 1 through n . The sum of all the digits of all the page numbers adds to 6951. What is n ?

Solutions should be submitted to Morgan Sherman:

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before next Thursday. Those with correct and complete solutions will have their names listed on the puzzle's web site (see below) as well as in next week's email announcement. Anybody is welcome to make a submission.

<http://www.calpoly.edu/~sherman1/puzzleoftheweek>

Solution: The book has 605 pages.

First, the sum of the digits 0 through 9 is $0 + 1 + 2 + \dots + 9 = \frac{9 \cdot 10}{2} = 45$. From there we see that the sum of all the digits in the numbers 0 through 99 is $45 + (10 + 45) + (20 + 45) + \dots + (90 + 45) = 450 + 10 \cdot 45 = 900$. Then we can work out the following table:

Pages	Sum of all digits
0 - 99	900
1 - 199	$100 + 900 = 1000$
2 - 299	$200 + 900 = 1100$
3 - 399	$300 + 900 = 1200$
4 - 499	1300
5 - 599	1400
Total:	6900

Then pages 600 through 605 contribute another 51 to the total sum, giving the answer above.