Cal Poly Department of Mathematics

Puzzle of the Week
Oct 16 - 22, 2014

A large Calculus book has pages numbered 1 through \( n \). The total number of digits in all the page numbers is 6949. How many pages are in the book?

Solutions should be submitted to Morgan Sherman:

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before next Thursday. Those with correct and complete solutions will have their names listed on the puzzle’s web site (see below) as well as in next week’s email announcement. Anybody is welcome to make a submission.

http://www.calpoly.edu/~sherman1/puzzleoftheweek

Solution: There are 2014 pages.

We note that there are:
9 numbers (1-9) with 1 digit \( 1 \times 9 \)
90 numbers (10-99) with 2 digits \( 2 \times 90 \)
900 numbers (100-999) with 3 digits \( 3 \times 900 \)
9000 numbers (1000-9999) with 4 digits \( 4 \times 9000 \)

etc.

So the number \( N \) of pages must be more than 999 but less than 9999. Then we have:

\[
1 \times 9 + 2 \times 90 + 3 \times 900 + 4 \times (9999 - N) = 6949
\]

which leads to the \( N = 2014 \).