Determine, with justification, which is larger: \((1.000001)^{1,000,000}\) or 2.

Note: A computer algebra system may be able to give many decimal places of the first number; the point here is to justify the result without this assistance.

Solution:  In fact \((1.000001)^{1,000,000} > 2\).

The quickest solution is probably to apply the binomial theorem. For any integer \(N \geq 2\):

\[
\left(1 + \frac{1}{N}\right)^N = \sum_{i=0}^{N} \binom{N}{i} \frac{1}{N^i} = 1 + \binom{N}{1} \frac{1}{N} + \text{positive terms} = 2 + \text{positive terms} > 2
\]