Cal Poly Department of Mathematics

Puzzle of the Week

Oct 9 - 15, 2014

Determine, with justification, which is larger: $(1.000001)^{1,000,000}$ or 2.

Note: A computer algebra system may be able to give many decimal places of the first number; the point here is to justify the result without this assistance.

Solutions should be submitted to Morgan Sherman:

Dept. of Mathematics, Cal Poly
Email: sherman1 -AT- calpoly.edu
Office: bldg 25 room 310

before next Thursday. Those with correct and complete solutions will have their names listed on the puzzle's web site (see below) as well as in next week's email announcement. Anybody is welcome to make a submission.

http://www.calpoly.edu/~sherman1/puzzleoftheweek

Solution: In fact $(1.000001)^{1,000,000} > 2$.

The quickest solution is probably to apply the binomial theorem. For any integer $N \geq 2$:

$$\left(1+\frac{1}{N}\right)^N = \sum_{i=0}^{N} \binom{N}{i} \frac{1}{N^i} = 1 + \binom{N}{1}\frac{1}{N} + (\text{positive terms}) \;=\; 2 + (\text{positive terms}) > 2$$