Cal Poly Department of Mathematics

Puzzle of the Week

Feb 13-19, 2014

If n is an integer can both $n+3$ and n^2+3 be perfect cubes?

Solutions should be submitted to Morgan Sherman:

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before next Thursday. Those with correct and complete solutions will have their names listed on the puzzle's web site (see below) as well as in next week's email announcement. Anybody is welcome to make a submission.

http://www.calpoly.edu/~sherman1/puzzleoftheweek

Solution: No, both $n+3$ and n^2+3 cannot be perfect cubes.

Suppose they are. Then their product is as well. However

$$(n+3)(n^2+3) = n^3+3n^2+3n+9 = (n+1)^3+8$$

and so we would have two perfect cubes differing by exactly 8. So now you have to verify that the only pairs of perfect cubes which differ by 8 are $(0,8)$ and $(-8,0)$. In each case a direct check shows that neither of these work.