## Cal Poly Department of Mathematics

# Puzzle of the Week

Nov 21 - Dec 2, 2013

What happens to the sum

$$\frac{1}{1^2+2013n^2}+\frac{2}{2^2+2013n^2}+\frac{3}{3^2+2013n^2}+\ldots+\frac{n}{n^2+2013n^2}$$

as $n \to \infty$?

*Solutions should be submitted to Morgan Sherman:*


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*before ~~next Thursday~~ December 2nd. Those with correct and complete solutions will have their names listed on the puzzle's web site (see below) as well as in next week's email announcement. Anybody is welcome to make a submission.*

http://www.calpoly.edu/~sherman1/puzzleoftheweek

Solution: The sum tends to $\frac{1}{2}\log\frac{2014}{2013}$.

The sum equals

$$\sum_{k=1}^{n}\frac{k}{k^2+2013n^2}=\sum_{k=1}^{n}\frac{(\frac{k}{n})}{(\frac{k}{n})^2+2013}\,\frac{1}{n}$$

which is a Riemann sum for the function $f(x)=\frac{x}{x^2+2013}$ on the interval $[0,1]$ (with $n$ rectangles of equal widths, and heights given by the function value at the right endpoint of each subinterval). Therefore

$$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{k}{k^2+2013n^2}=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{(\frac{k}{n})}{(\frac{k}{n})^2+2013}\,\frac{1}{n}=\int_0^1\frac{x}{x^2+2013}\,dx=\frac{1}{2}\int_{2013}^{2014}\frac{1}{u}\,du$$

(where $u=x^2+2013$) which evaluates to the above solution.