Cal Poly Department of Mathematics

Puzzle of the Week

October 24-30, 2013

Find all the prime numbers that are in the infinite sequence:

$$1,\ 10001,\ 100010001,\ 1000100010001,\ \ldots$$

or prove that there are none.

Solutions should be submitted to Morgan Sherman:

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before next Thursday. Those with correct and complete solutions will have their names listed on the puzzle's web site (see below) as well as in next week's email announcement. Anybody is welcome to make a submission.

http://www.calpoly.edu/~sherman1/puzzleoftheweek

Solution:

There are no primes in the sequence. The $n+1$st term in the sequence is $1+10^4+10^8+\ldots+10^{4n}$. If $n=2k+1$ is odd then this factors directly:

$$1+10^4+10^8+\ldots+10^{4n}=(1+10^4)(1+10^8+10^{2\cdot 8}+\ldots+10^{8k})$$

so if $k>0$ this shows the number is composite. If $k=0$ then use just use $10001=73\times 137$.

If on the other hand $n=2k$ is even then we can use the geometric series formula:

$$1+10^4+10^8+\ldots+10^{4n}=\frac{(10^4)^{2k+1}-1}{10^4-1}=\left(\frac{10^{4m+2}-1}{10^2-1}\right)\left(\frac{10^{4m+2}+1}{10^2+1}\right)$$

and one checks these are both integers (the first term is $1+10^2+\ldots+10^{4m}$ and the second is $1-10^2+10^4-10^6+\ldots+10^{4m}$).