Cal Poly Department of Mathematics

Puzzle of the Week

October 17-23, 2013

The other day I walked by the math department chair's office and naively asked "Joe, what time is it?". Professor Borzellino looked at the clock on his wall and replied:

$$\text{"It is } \frac{\pi}{\operatorname{arccsc}(2\varphi)} \text{ o'clock"}.$$

What time was it?

Note: $\varphi = \frac{1+\sqrt{5}}{2}$ is the *golden ratio*. Also most computer algebra systems will instantly give the answer (as will simply walking into Joe's office); for a complete solution you must show how to calculate this by hand.

Solutions should be submitted to Morgan Sherman:

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before next Thursday. Those with correct and complete solutions will have their names listed on the puzzle's web site (see below) as well as in next week's email announcement. Anybody is welcome to make a submission.

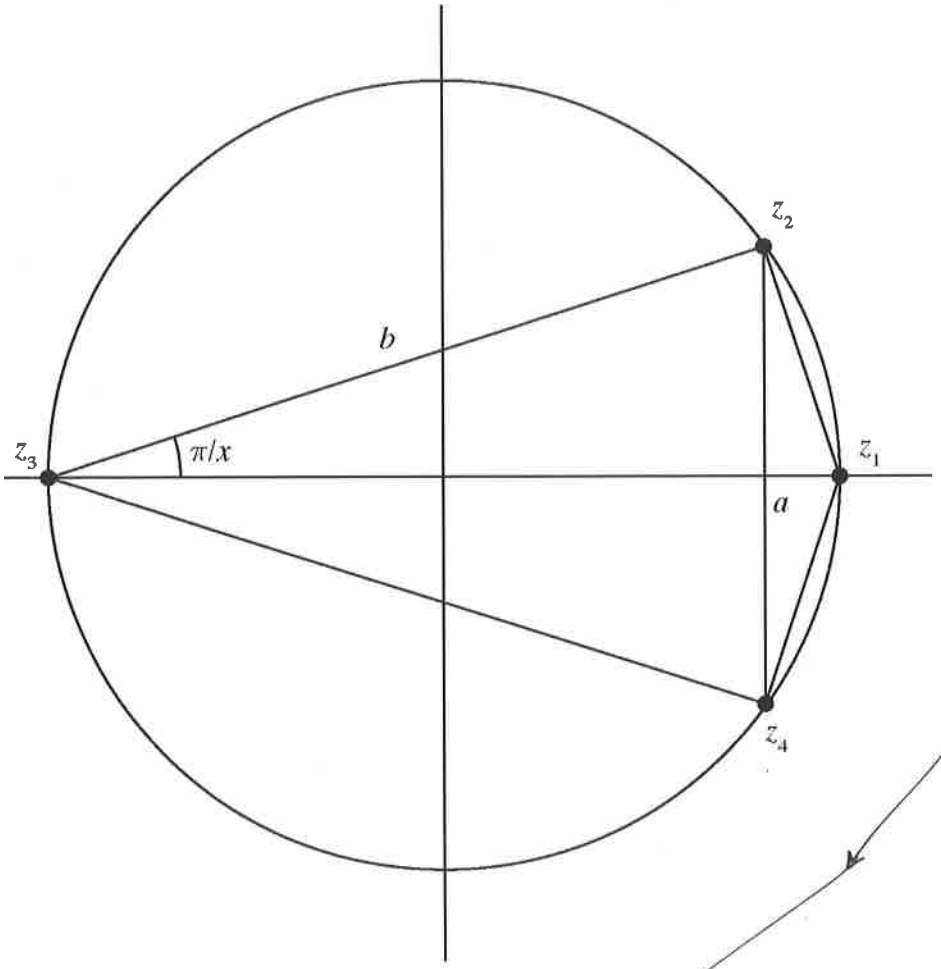
http://www.calpoly.edu/~sherman1/puzzleoftheweek

Solution: It was 10 o'clock.

A number of different types of solutions came in, many using nice geometric arguments. First note that $\frac{\pi}{\operatorname{arccsc}(2\varphi)} = 10 \iff \sin(\frac{\pi}{10}) = \frac{1}{1+\sqrt{5}}$. Now one can show this using repeated angle sum formulas. This brute force calculation is not pretty but works.

Some more geometric solutions were suggested as well, many making use of a pentagon and the fact that a regular pentagon with side length equal to 1 will have the length of its diagonal (distance from a vertex to a non-adjacent vertex) equal to φ.

The shortest "proof" that came in though probably belonged to Erin Pearse, who submitted the picture on the next page. I'll leave it to you to work out the details:



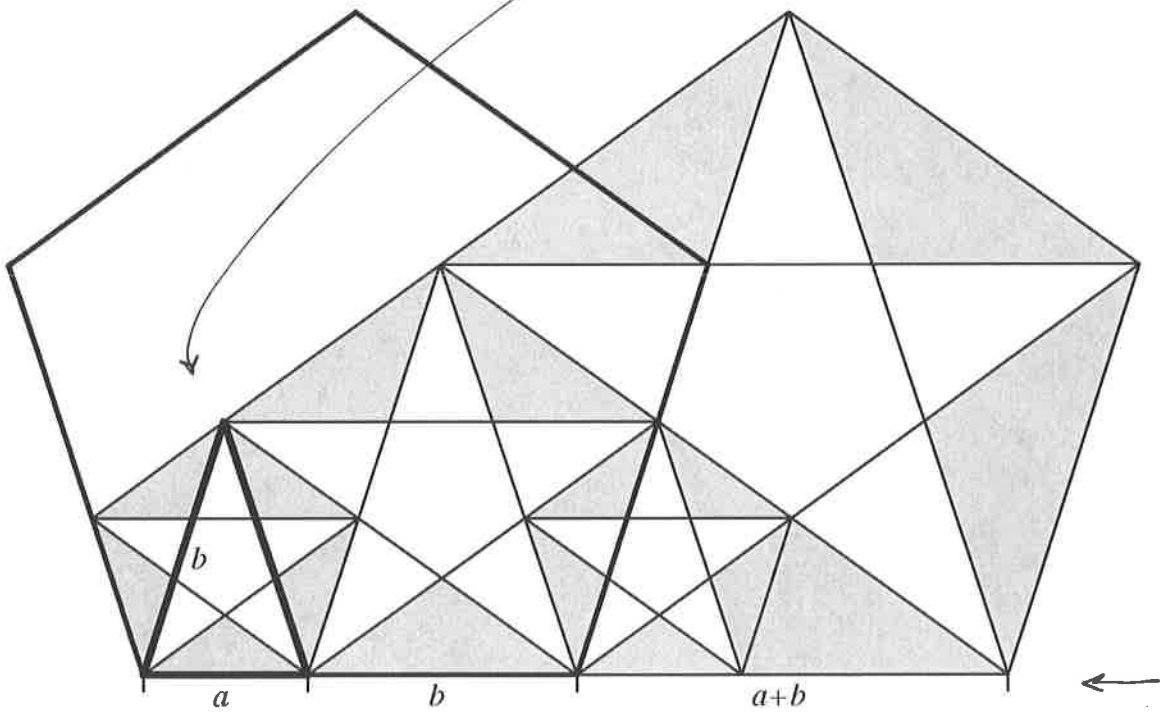
$$\frac{\pi}{\operatorname{arccsc}(2\varphi)} = x$$

$$\Leftrightarrow \quad \frac{1}{2\varphi} = \sin\frac{\pi}{x} = \frac{a/2}{b}$$

$$\Leftrightarrow \quad \varphi = \frac{b}{a}$$

$\frac{\pi}{x} = \frac{\pi}{10}$ by diagram

$$\Leftrightarrow \quad \boxed{x = 10}$$



← Clearly, $\frac{b}{a} = \frac{a+b}{b}$,

so $\frac{b}{a} = \varphi$