Cal Poly Department of Mathematics

Puzzle of the Week

May 16-22, 2013

A geometric series has as its first term a positive integer, and its common ratio is the reciprical of a positive integer. If the series sums to 5 what is the sum of the first two terms?

Solutions should be submitted to Morgan Sherman:

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before next Thursday. Those with correct and complete solutions will have their names listed on the puzzle's web site (see below) as well as in next week's email announcement. Anybody is welcome to make a submission.

http://www.calpoly.edu/~sherman1/puzzleoftheweek

Solution: The sum of the first two terms is $\frac{24}{5}$.

If a is the first term and $1/b$ is the common ratio then the standard formula for the sum of a geometric series gives us

$$\frac{a}{1-\frac{1}{b}} = 5$$

so that $a = 5 - \frac{5}{b}$. Since a and b are positive integers this is only satisfied if $b = 5$, in which case $a = 4$. So the first two terms sum to $a + \frac{a}{b} = 4 + \frac{4}{5}$.