Cal Poly Department of Mathematics

Puzzle of the Week

Feb 7 - 13, 2013

Let N be the number formed by concatenating the decimal expansions of the numbers 16^{2013} and 625^{2013}. How many digits does N have?

Solutions should be submitted to Morgan Sherman:

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before next Thursday. Those with correct and complete solutions will have their names listed on the puzzle's web site (see below) as well as in next week's email announcement. Anybody is welcome to make a submission.

http://www.calpoly.edu/~sherman1/puzzleoftheweek

Solution: There are 8053 total digits.

There are positive integers a, b such that

$$10^a < (2^4)^{2013} < 10^{a+1}, \quad 10^b < (5^4)^{2013} < 10^{b+1}.$$

We see that the number we are looking for is $(a+1)+(b+1) = (a+b+2)$. Now, multiplying the inequalities above we have

$$10^{a+b} < 10^{4\cdot 2013} < 10^{a+b+2}$$

so that $4 \cdot 2013 = a + b + 1$. Therefore $a + b + 2 = 4 \cdot 2013 + 1 = 8053$.