Cal Poly Department of Mathematics

Puzzle of the Week

Jan 17 - 23, 2013

From Kent Morrison:

I would like to cook in a (circular) pan of radius r three slices of french toast, each a square of side length s. What is the smallest value of r for which I can put all three slices in at once, with no overlap? [You may assume the squares are all parallel to each other, i.e. all oriented at the same angle.]

Solutions should be submitted to Morgan Sherman:

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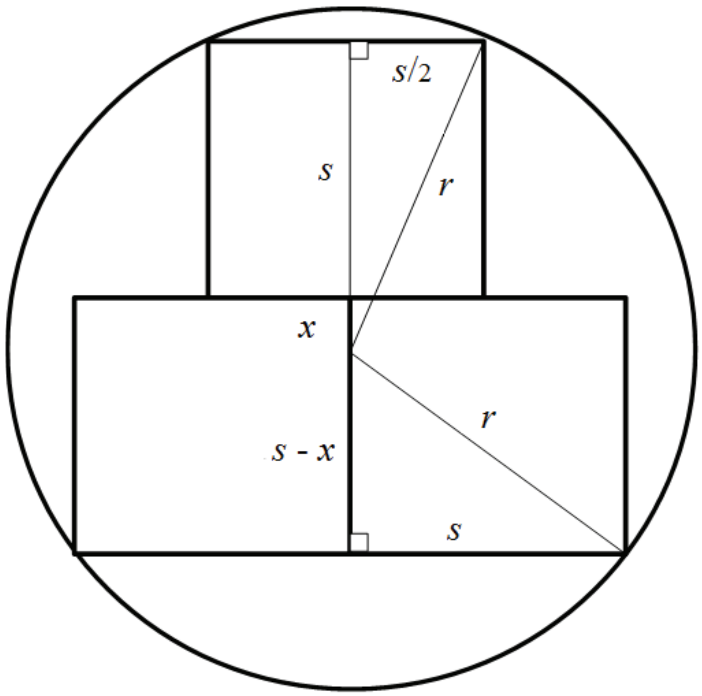
before next Thursday. Those with correct and complete solutions will have their names listed on the puzzle's web site (see below) as well as in next week's email announcement. Anybody is welcome to make a submission.

http://www.calpoly.edu/~sherman1/puzzleoftheweek

Solution: The smallest radius is $r = \frac{5\sqrt{17}}{16}s$. Below is Tom O'Neil's nice solution, which he has kindly allowed me to reproduce:

Determine the smallest circle of radius r that can contain three nonoverlapping squares of side length s. We place the squares inside the circle as shown in the following figure. Here the center of the circle will lie on the common side of the two adjacent squares and x units below their upper sides.



Using the Pythagorean theorem on the two right triangles gives us the two equations

$$r^2 = s^2 + (s-x)^2 = 2s^2 - 2xs + x^2 \text{ and}$$
$$r^2 = \left(\frac{s}{2}\right)^2 + (s+x)^2 = \frac{5s^2}{4} + 2xs + x^2.$$

Subtracting yields

$$\frac{3s^2}{4} - 4xs = 0$$

or eqivelently

$$x = \frac{3s}{16}.$$

Using this value for x in the first equation gives us

$$r^2 = s^2 + \left(s - \frac{3s}{16}\right)^2 = s^2 + \left(\frac{13s}{16}\right)^2 = s^2 + \frac{169s^2}{256} = \frac{425s^2}{256}$$

which using only the positive solution gives us

$$r = \frac{5\sqrt{17}}{16}s$$

as the smallest radius of an encompassing circle.