On a given circle there are two fixed points $A, B$ not diametrically opposite each other. Let $X$ denote a variable point on the circle, and let $X'$ be the point diametrically opposite $X$. Determine the locus of points $AX \cap BX'$ (i.e. the point of intersection of the lines $AX$ and $BX'$) as $X$ varies around the circle.

Solutions should be submitted to Morgan Sherman:

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before next Thursday. Those with correct and complete solutions will have their names listed on the puzzle’s web site (see below) as well as in next week’s email announcement. Anybody is welcome to make a submission.

http://www.calpoly.edu/~sherman1/puzzleoftheweek

**Solution:** I found this problem in Larson’s book “Problem-Solving Through Problems”, which is an excellent guide to techniques of problem-solving.

The locus of points will be a circle which passes through $AB$. To see this, and see which circle, let $P$ be the point of intersection and assume first that it lies inside the circle. Then note that these $P$ all satisfy that angle($APB$) = $90^\circ + \frac{1}{2}$(Arc $AB$). Similarly you can check that when $P$ lies outside the circle this angle becomes angle($APB$) = $90^\circ - \frac{1}{2}$(Arc $AB$). I’ll leave it to you to verify that these points (when $P$ lies inside versus outside the circle) all lie on the same circle.