

Cal Poly Department of Mathematics

Puzzle of the Week

Oct 4 - Oct 10, 2012

From Tom O'Neil:

Fix an $a > 0$. If you rotate the curve $y = e^{-x/a} \sin x$, $x \geq 0$, about the x -axis then you get a picture which looks like an infinite string of beads. What is the total volume of all those beads?

Solutions should be submitted to Morgan Sherman:

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before next Tuesday. Those with correct and complete solutions will have their names listed on the puzzle's web site (see below) as well as in next week's email announcement. Anybody is welcome to make a submission.

<http://www.calpoly.edu/~sherman1/puzzleoftheweek>

Solution: The total volume is $\frac{\pi a^3}{4 a^2 + 1}$. This can be calculated using the standard formula for the volume of a revolution about the x -axis:

$$\text{Volume} = \int_0^{\infty} \pi y^2 dx = \pi \int_0^{\infty} e^{-\frac{2x}{a}} \sin^2 x dx$$

which can be computed using a variety of techniques.

For a more exciting solution one can calculate the volume of the n th "bead", then sum an infinite series to obtain the same result.