Cal Poly Department of Mathematics

Puzzle of the Week

May 22-28, 2012

This week's "puzzle" will be a little different. It won't require a proof or even any calculation. Just try the following: Take 21 objects (they can be coins, cards, pieces of chalk, books, rubies, whatever you like) and split them into piles of various sizes however you like. Now take one object from each pile and create a new pile with these; repeat this. What happens?

Notes: Just describe the resulting phenomenon (you'll know it when you see it), don't prove it (I believe a proof is actually fairly difficult). The above also works if you start with $1, 3, 6, 10, 15, 21, 28, \ldots$ objects, but not necessarily with other numbers.

Solutions should be submitted to Morgan Sherman:

Dept. of Mathematics, Cal Poly
Email: sherman1 -AT- calpoly.edu
Office: bldg 25 room 310

before next Tuesday. Those with correct and complete solutions will have their names listed on the puzzle's web site (see below) as well as in next week's email announcement. Anybody is welcome to make a submission.

http://www.calpoly.edu/~sherman1/puzzleoftheweek

<u>Solution:</u> The objects will ultimately arrange themselves in piles of 1, 2, 3, 4, 5, and 6, and this pattern repeats indefinitely. In general, if we have T_n objects, where $T_n = 1 + 2 + \ldots + n = n(n+1)/2$ is the nth *triangular number*, then the objects will arrange themselves into piles of sizes $1, 2, \ldots, n$.

This game is called "Bulgarian Solitare" and was poularized by a column of the late great Martin Gardner[1], which itself was inspired by an article of Jørgen Brandt[2]. For a proof, you may want to see the *Monthly* article by Ethan Akin and Mortin Davis[3].

[1] M. Gardner, Mathematical games, Scientific American, 249 (1983) 12-21

[2] J. Brandt, Cycles of partitions, Proc. Amer. Math. Soc., 85 (1982) 483-486

[3] E. Akin and M. Davis, Bulgarian Solitaire, The American Mathematical Monthly, Vol. 92, No. 4 (April 1985), pp. 237-250