Cal Poly Department of Mathematics

Puzzle of the Week

May 15-21, 2012

Consider the n-dimensional cube $C_n = [-2,2]^n$ (that is all points in $\mathbb{R}^n$ all of whose coordinates are between -2 and 2). At each of the points whose coordinates are each either ± 1 place a unit ball (there will be 2^n of them). Now let B_n denote the largest ball centered at the origin not intersecting any of these. Calculate

$$\lim_{n\to\infty} \frac{\mathrm{Vol}(B_n)}{\mathrm{Vol}(C_n)}.$$

Note that you will need to look up the formula for the volume of a ball of radius R in n-dimensional space.

[By way of example, if $n = 2$ then we will place a unit disk at each of $(1,1), (1,-1), (-1,1), (-1,-1)$. We calculate that B_2 will have radius $\sqrt{2}-1$ and $\frac{\mathrm{Vol}(B_2)}{\mathrm{Vol}(C_2)} = \frac{\pi(\sqrt{2}-1)^2}{16} \approx 0.034$]

Solutions should be submitted to Morgan Sherman:

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before next Tuesday. Those with correct and complete solutions will have their names listed on the puzzle's web site (see below) as well as in next week's email announcement. Anybody is welcome to make a submission.

http://www.calpoly.edu/˜sherman1/puzzleoftheweek