Cal Poly Department of Mathematics

Puzzle of the Week

May 10-15, 2012

Evaluate the sum

$$\frac{1}{1\cdot 2\cdot 3}+\frac{1}{3\cdot 4\cdot 5}+\frac{1}{5\cdot 6\cdot 7}+\dots$$

Solutions should be submitted to Morgan Sherman:

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before next Tuesday. Those with correct and complete solutions will have their names listed on the puzzle's web site (see below) as well as in next week's email announcement. Anybody is welcome to make a submission.

http://www.calpoly.edu/˜sherman1/puzzleoftheweek

<u>*Solution:*</u> The series converges to $-\frac{1}{2}+\log 2$.

The series converges by comparison to the p-series $\sum \frac{1}{n^3}$. Using partial fractions we find that

$$\frac{1}{k(k+1)(k+2)}=\frac{1/2}{k}-\frac{1}{k+1}+\frac{1/2}{k+2}$$

and therefore

$$\begin{aligned}\frac{1}{1\cdot 2\cdot 3}+\frac{1}{3\cdot 4\cdot 5}+\frac{1}{5\cdot 6\cdot 7}+\dots &=\\ &=\left(\frac{1/2}{1}-\frac{1}{2}+\frac{1/2}{3}\right)+\left(\frac{1/2}{3}-\frac{1}{4}+\frac{1/2}{5}\right)+\left(\frac{1/2}{5}-\frac{1}{6}+\frac{1/2}{7}\right)+\dots\\ &=-\frac{1}{2}+\left(1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\dots\right)\end{aligned}$$

which equals the above solution using the known sum of the alternating harmonic series.