Puzzle of the Week
April 24-30, 2012

A convex octagon is inscribed in a circle and has four consecutive sides of length 3 and four consecutive sides of length 4. Find its area and express your answer in the form $a + b\sqrt{c}$ with $a, b, c$ positive integers.

Solutions should be submitted to Morgan Sherman:
Dept. of Mathematics, Cal Poly
Email: sherman1 -AT- calpoly.edu
Office: bldg 25 room 310

before next Tuesday. Those with correct and complete solutions will have their names listed on the puzzle’s web site (see below) as well as in next week’s email announcement. Anybody is welcome to make a submission.

http://www.calpoly.edu/~sherman1/puzzleoftheweek

Solution: The octagon has area $25 + 24\sqrt{2}$.

This problem appears on the 1978 Putnam Exam as B-1 (with slightly different edge lengths). There are many ways of approaching the problem (and I received a number of different correct solutions). One simple way is to recognize that the octagon has the same area as one whose sides alternate lengths 3 and 4 (convince yourself of this!):

Now it is a simple matter to compute the area and arrive at the value above.