Cal Poly Department of Mathematics

Puzzle of the Week

Feb 9-15, 2012

A polynomial has the form $p(x) = x^{2012} - 2000x^{2011} + \ldots + 13$. If all of its roots are integers find them, listed with their multiplicities.

Solutions should be submitted to Morgan Sherman:

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before next Thursday. Those with correct and complete solutions will have their names listed on the puzzle's web site (see below) as well as in next week's email announcement. Anybody is welcome to make a submission.

http://www.calpoly.edu/~sherman1/puzzleoftheweek

Solution: The roots are: 13; 1 with multiplicity 1999; and -1 with multiplicity 12.

If we write the polynomial in factored form $p(x) = (x - r_1) \cdot (x - r_2) \cdot \ldots \cdot (x - r_{2012})$ and expand we find that $\prod r_i = 13$ and $\sum r_i = 2000$. Since all the roots are integers we find from the first equality that exactly one root, say r_{2012} must be ± 13, and the remaining roots are all ± 1. If $r_{2012} = -13$ then the second equality becomes impossible. On the other hand if $r_{2012} = +13$ then it becomes possible with the above choices for the multiplicities of $+1$ and -1.