

Cal Poly Department of Mathematics

Puzzle of the Week

Jan 12-18, 2012

Determine, with justification, which is larger:

$$2011^{2012} \quad \text{or} \quad 2012^{2011}$$

Solutions should be submitted to Morgan Sherman:

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before next Friday. Those with correct and complete solutions will have their names listed on the puzzle's web site (see below) as well as in next week's email announcement. Anybody is welcome to make a submission.

<http://www.calpoly.edu/~sherman1/puzzleoftheweek>

Solution:

2011^{2012} is larger.

In fact we'll show that $a^b > b^a$ whenever $b > a \geq e$. Since a, b are positive and $\log x$ is an increasing function we see that $a^b > b^a$ is equivalent to $b \log a > a \log b$. But this is equivalent to $\frac{\log a}{a} > \frac{\log b}{b}$. So we'll be done if we show that $f(x) = \frac{\log x}{x}$ is a decreasing function for $x > e$. However

$$f'(x) = \frac{1 - \log x}{x^2} < 0 \text{ whenever } x > e.$$